SIAM Conference on Applied Algebraic Geometry (AG15) August 3-7, 2015 at Daejeon (Korea)
MS64: Recent developments in Geometric and Algebraic Methods in Economics (Aug 7; 14:00-16:00)

Subtropical Convex Geometry as the Ricardian Theory of International Trade

Errors corrected on Aug. 9, 2015

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Contents

1. Ricardian Trade Economy
2. How I encountered tropical algebra
3. Some new ideas

Papers:

- Subtropical Convex Geometry as the Ricardian Theory of International Trade
  - This is the title of an incomplete book. Major part is uploaded in my ResearchGate page.
- International trade theory and exotic algebra
  - Published as Shiozawa, Y. 2015 International Trade Theory and Exotic Algebra. *Evolutionary and Institutional Economics Review* **15**(1): 177-212. A part of special issue, edited by E.-M. Feichtner and S. Settepanella.
- Request me for the papers: y@shiozawa.net

Backgound for Ricardian trade theory

- One of the oldest theories in economics
  - D. Ricardo 1817 Principles of Political Economy and Taxation. Chapter 7 On Foreign Trade.
- True and nontrivial
  - S. Ulam's question to P.Samuelson
  - Name a theory which is both true and nontrivial.
  - Samuelson's answer (after days): Ricardo's theory of comparative advantage

Ricardian theory of international trade

- *M*-country, *N*-commodity case
- Production: labor input economy
  - Can incorporate material inputs
  - No intermediate goods (no input trade) ⇒equivalent to labor input economy
  - Economy with input trade is named Ricardo-Sraffa trade economy.
- Best process (production technique) is physically determined.

A Ricardian trade economy

- Input coefficient matrix $A = (a_{ij})$
  - M-row N-column matrix
  - a_{ij} labor input coefficient in country *i* to produce product *j*
- labor power $\mathbf{q} = (q_i)$
- International value $\mathbf{v} = (\mathbf{w}, \mathbf{p})$
  - $\mathbf{w} = (w_i)$ wage rate for country *i*
  - $\mathbf{p} = (p_j)$ price for product *j*

PPS, admissible value

- Production possibility set (PPS)

  $P = \{\mathbf{y} \mid y_j = (\sum_i s_{ij}),\ \sum_j s_{ij}\, a_{ij} \leqq q_i,\ s_{ij} \geqq 0\ \forall i\}$

  ☆PPS is a polytope in $\mathbf{R}^N$.
- Admissible value $\mathbf{v} = (\mathbf{w}, \mathbf{p}) > \mathbf{0}$.

  $w_i\, a_{ij} \geqq p_j \quad \forall i, j$
- Main theorem

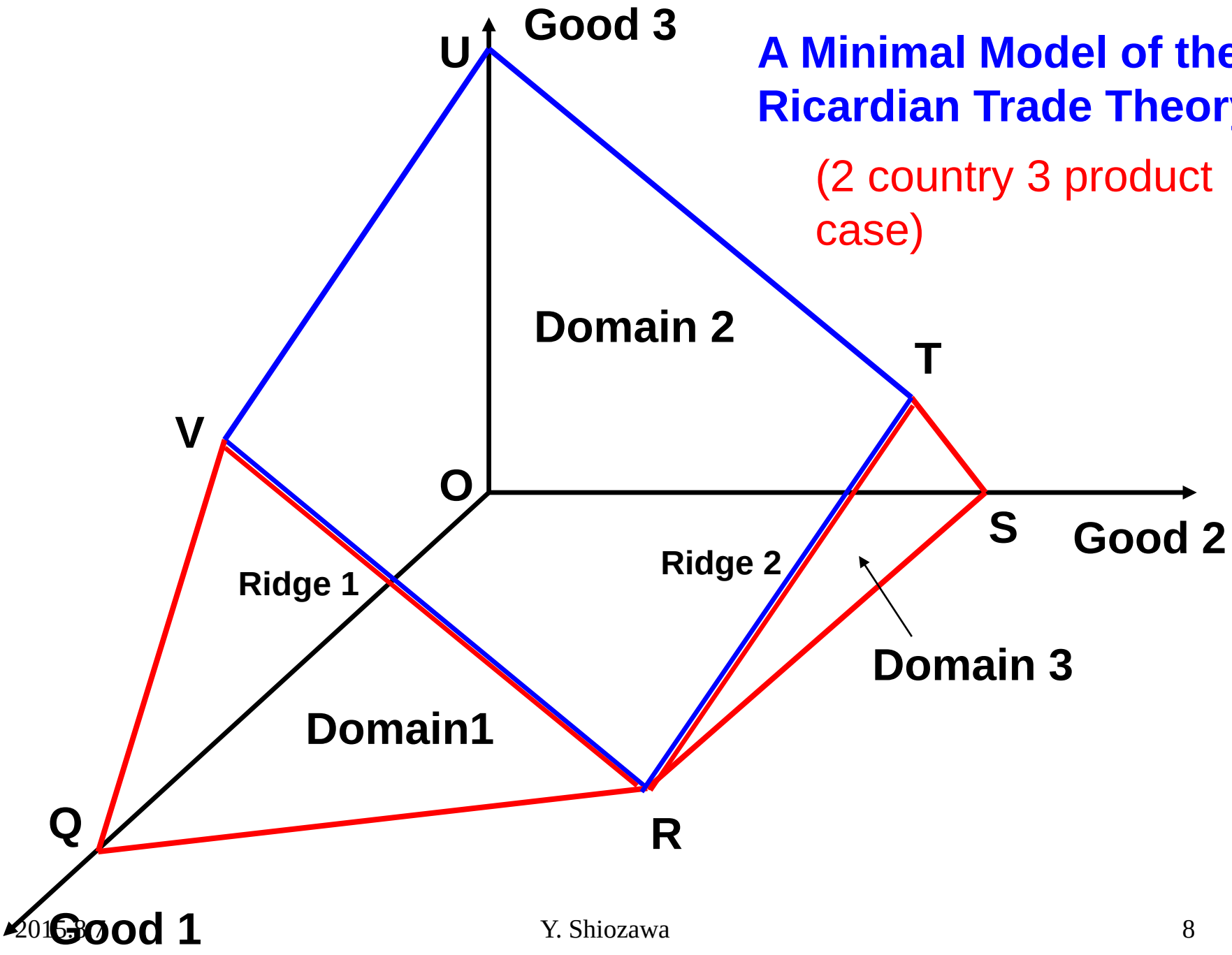
At each facet of PP set there exists an admissible international value with **p** perpendicular to the facet and satisfies equality:

$$\langle \mathbf{w}, \mathbf{q} \rangle = \langle \mathbf{p}, \mathbf{y} \rangle$$

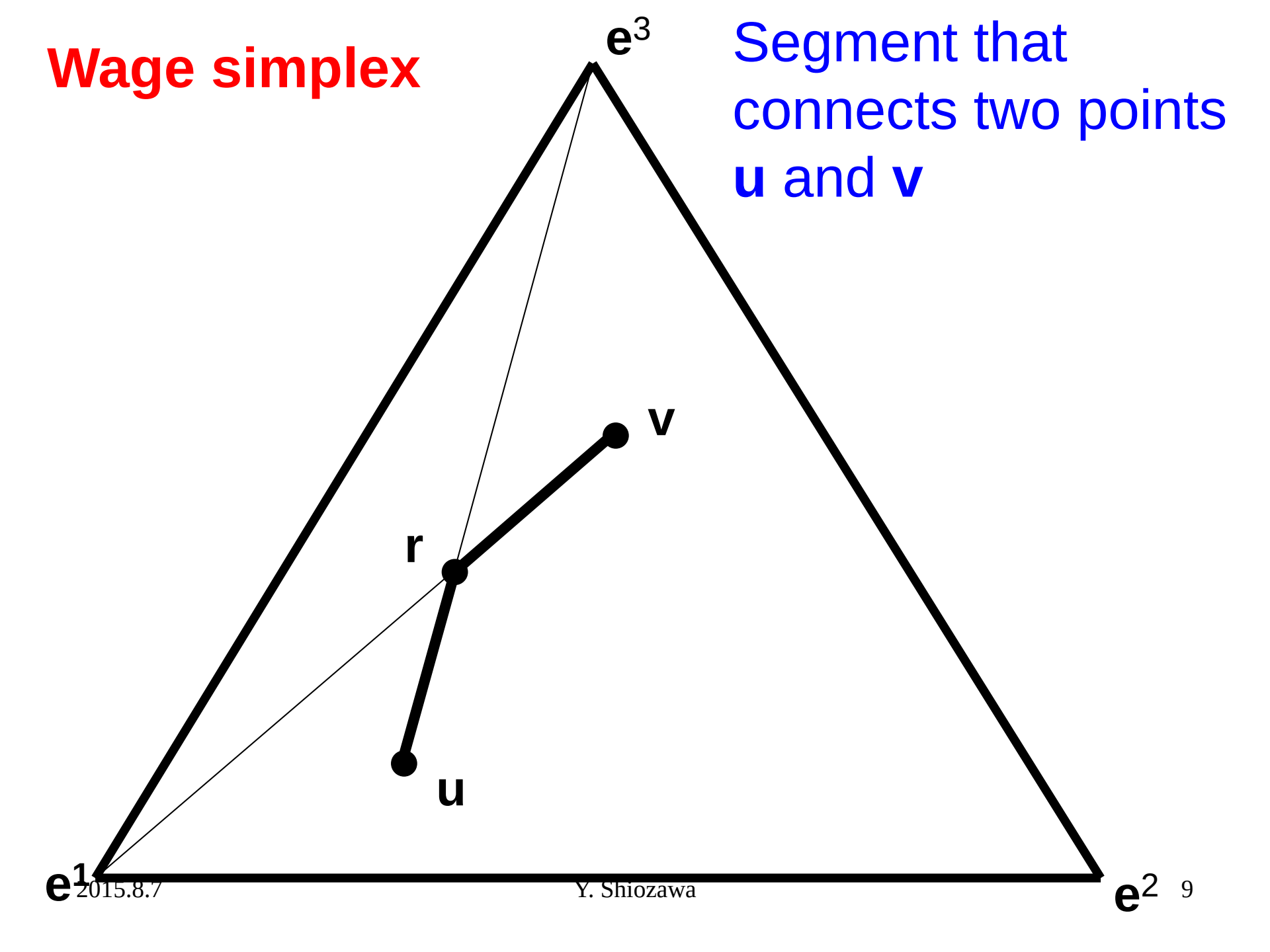
where **y** is a point in the facet.

A Minimal Model of the Ricardian Trade Theory

(2 country 3 product case)

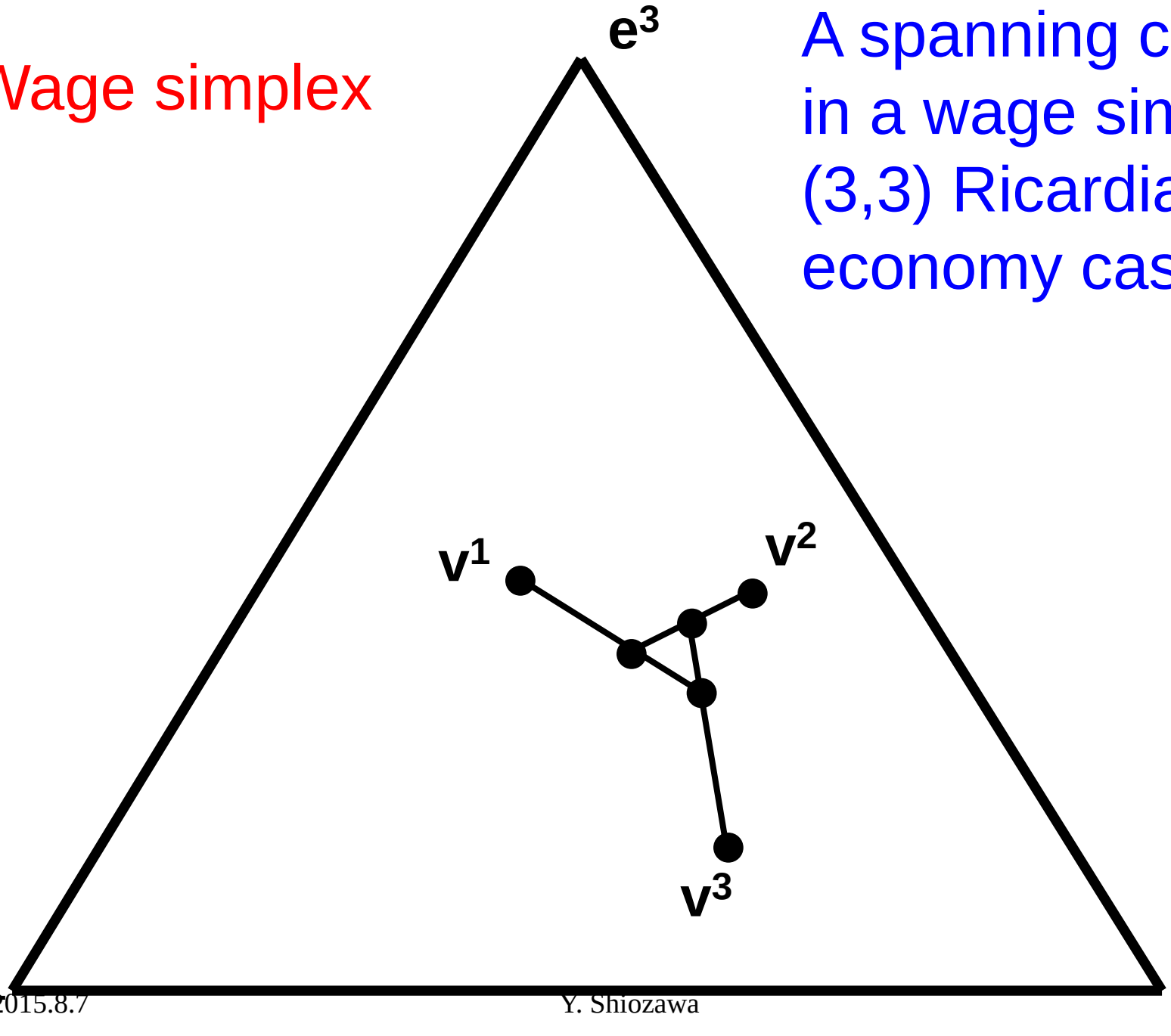


Wage simplex



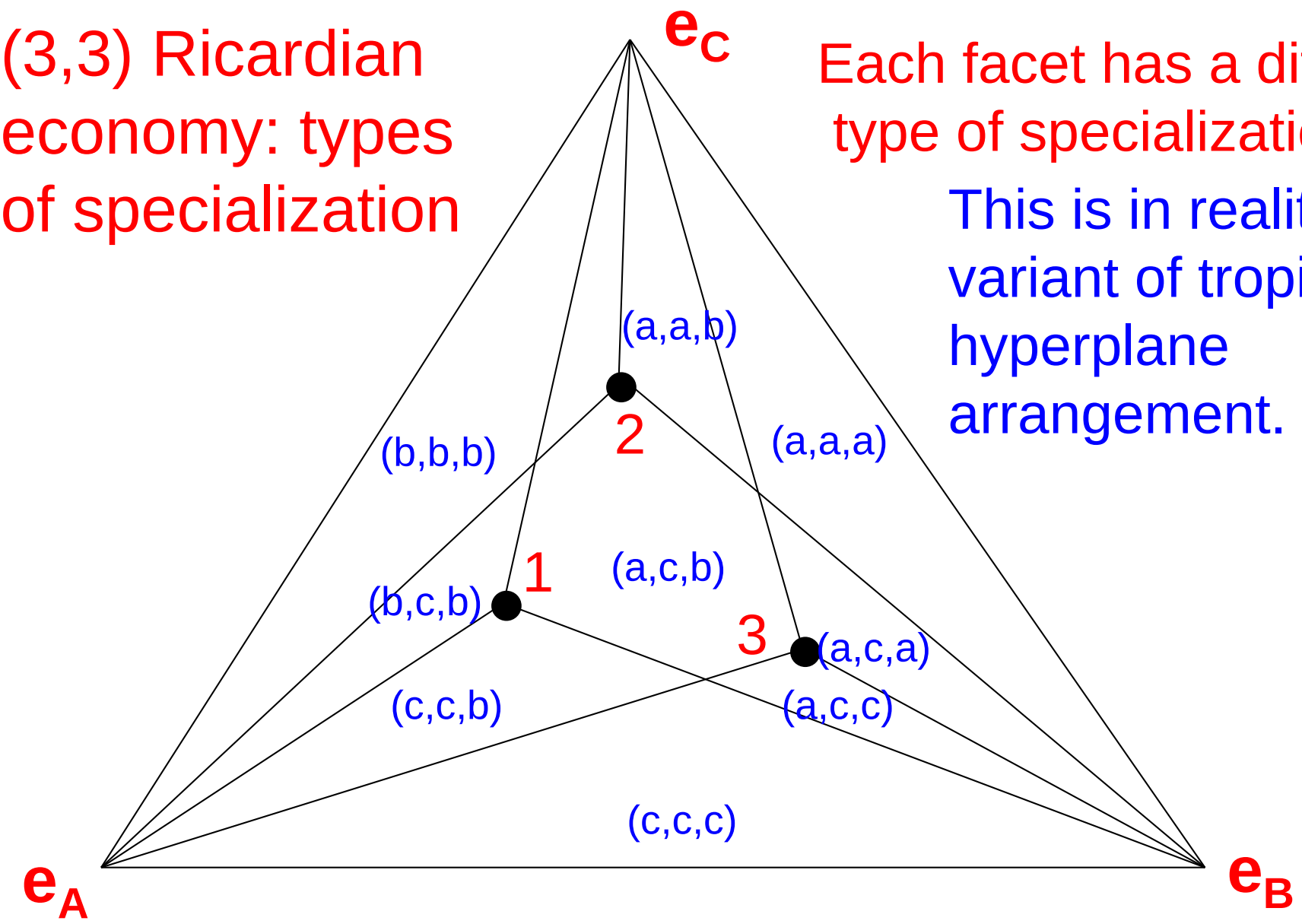
Segment that connects two points **u** and **v**

Wage simplex



A spanning core in a wage simplex: (3,3) Ricardian economy case

(3,3) Ricardian economy: types of specialization



Each facet has a differe type of specialization.

This is in reality a variant of tropical hyperplane arrangement.

Figure 4

An arrangement in $\mathbb{TP}^2$

Figure 1 (Ardila & Develin2004, p.3)

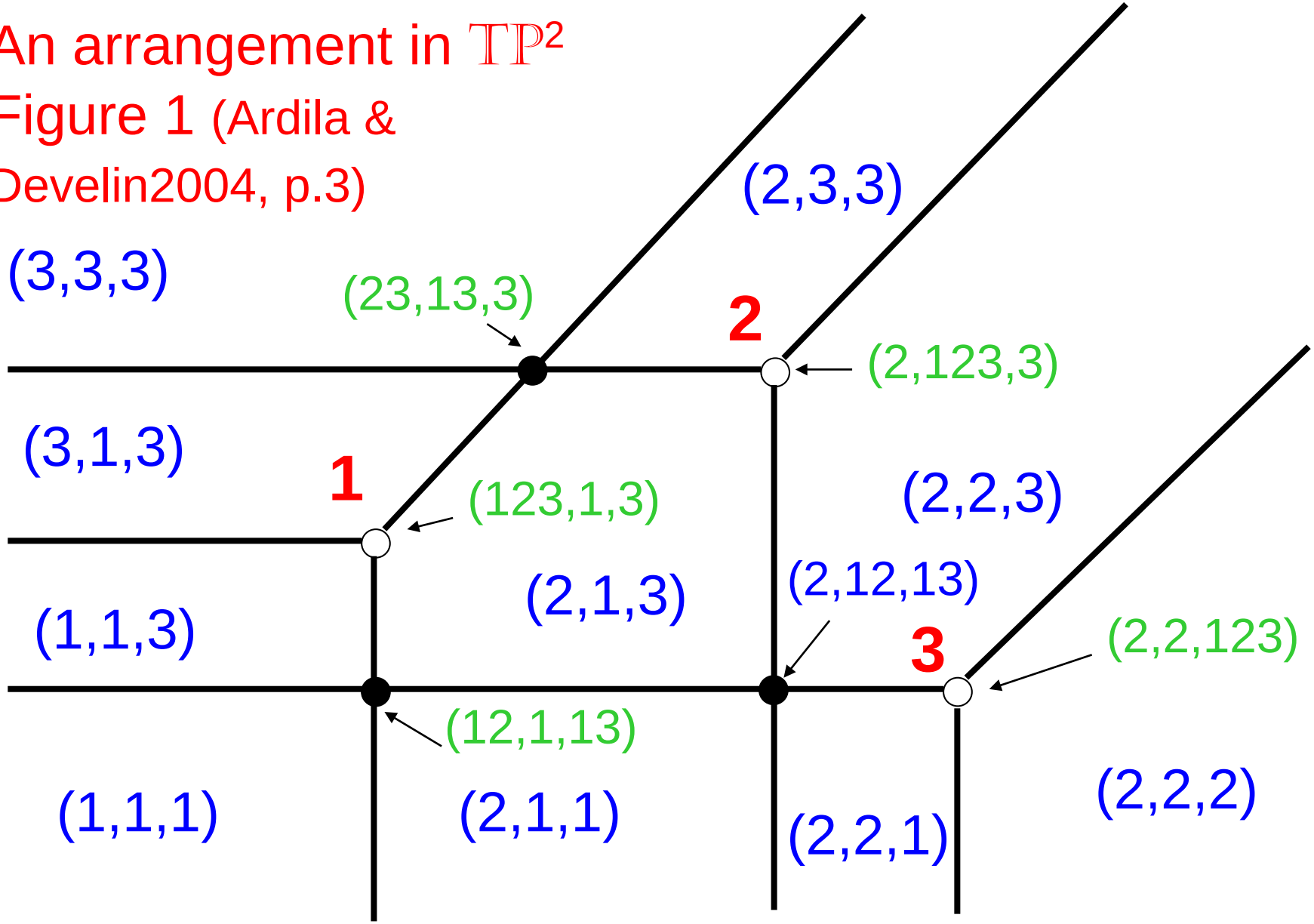
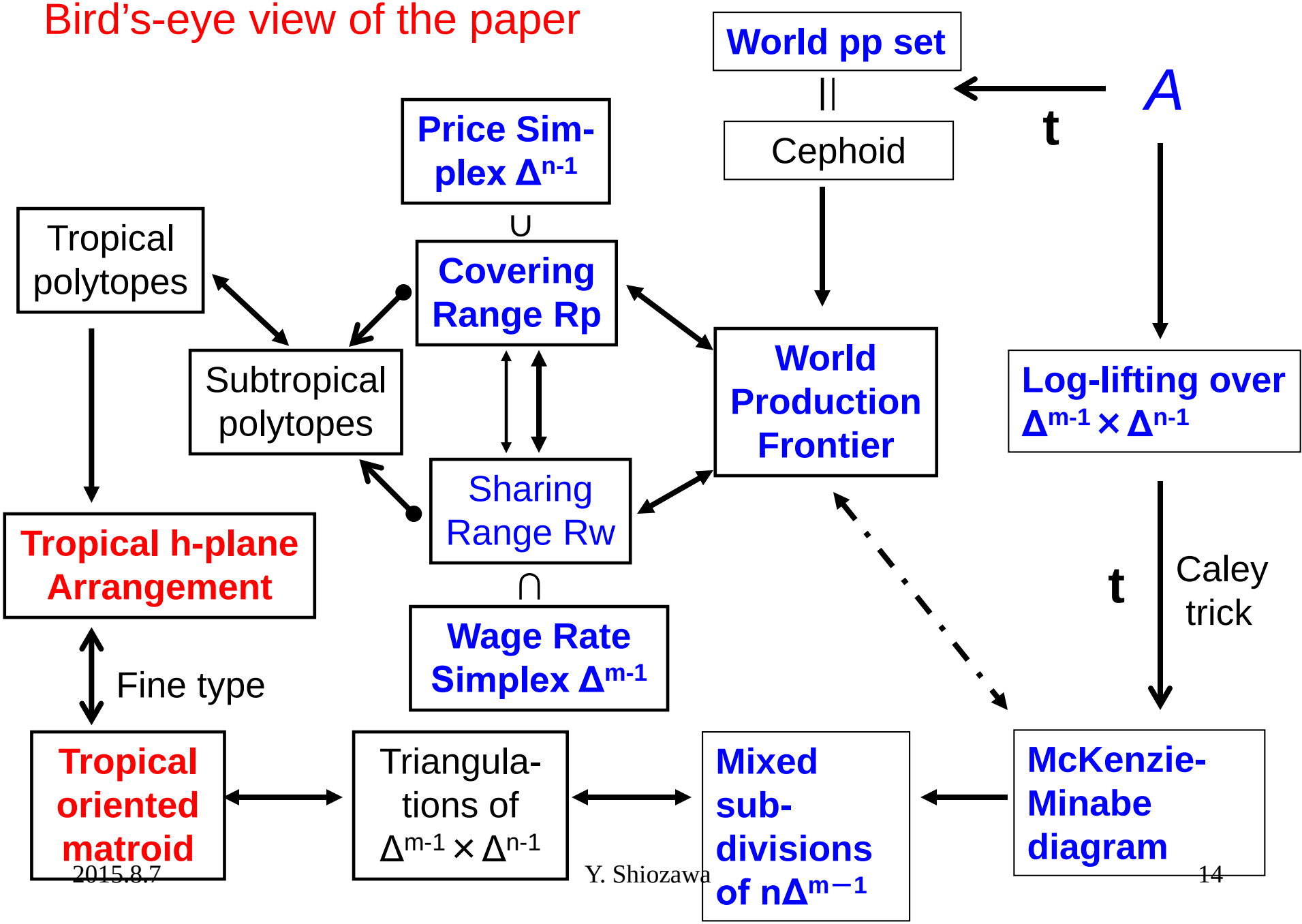


Figure 5

Subtropical algebra

- $a \oplus b = \min\{a, b\}$ $a \odot b = a \cdot b$
  - subtropical = a half way to the tropics.
- Ricardian trade theory
  - minimum, multiplication, Minkowski sum
  - a natural object for (sub)tropical analysis
- Matrix operation
  - $w \otimes A = \min_i w_i a_{ij}$ is comparable with p_j
  - **v** is admissible $\Leftrightarrow$ $w \otimes A = p$

Bird's-eye view of the paper



Some new ideas

- What happens in the interior of PPS?
  - Economically, this is to investigate underemployment.
  - This requires study admissible value independent of production point.
- Tropical oriented matroid:
  - a set of fine types (competitive type associated to an admissible value)

Necessary labor set

- A and d are given;
- L = { q | $q_i = (\sum_j s_{ij}\, a_{ij})_i$, $\sum_i s_{ij} = d_j$}
- An admissible value gives upper facet.
- An anti-admissible value gives lower facet. $w_{ij}\, a_{ij} \leqq p_j \quad \forall \tau=(i,j)$.
- Other values: mixed value
  - $\exists I,j \;\; wij\; aij < pj$ and $\exists h,k\; w_{hk}\, a_{hk} > p_k$

	使用労働量	第1財	第2財	第3財
A国	uA	1	2	1
B国	uB	3	1	1
両国の総産出量d		1	2	3

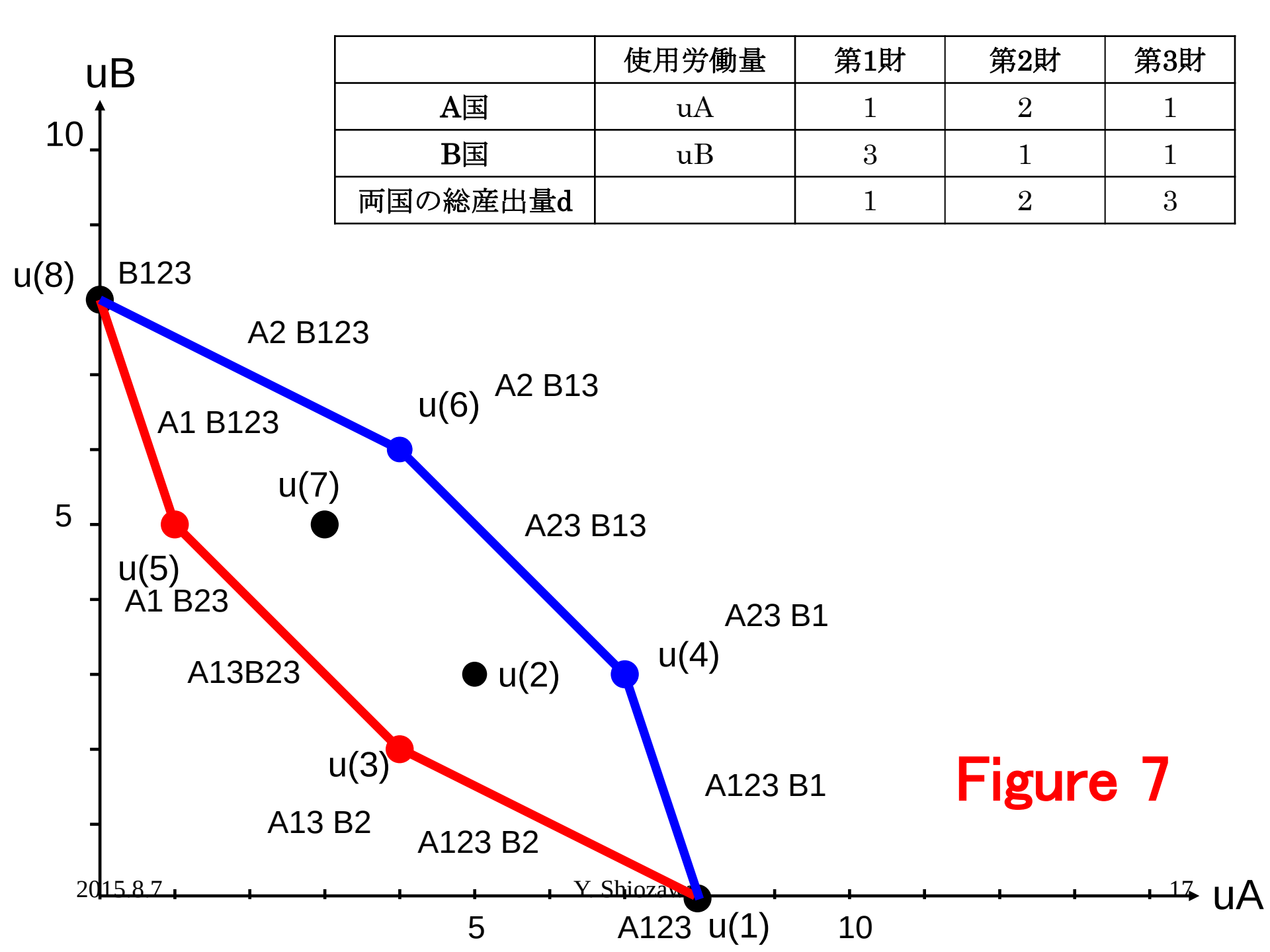


Figure 7

value ⇔ competitive type

- $A = (a_{ij})$ is given.

  $\mathbf{v} = (\mathbf{w}, \mathbf{p}) \Rightarrow T = \{\tau=(i,j) | \, w_i a_{ij} = p_j \}$
- T: (M,N) bipartite graph $T \in K_{M,N}$
- T: spanning tree
  - spanning (edges cover all countries and goods)
  - no cycle (no cyclic chain of edges)
  - In (2,3) trade economy, there are 12 different spanning trees. See the next sheet.

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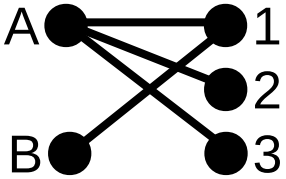

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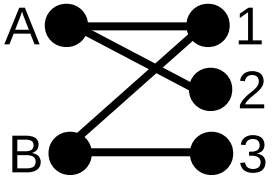

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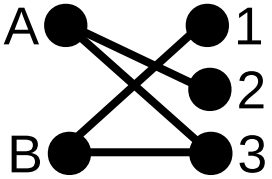

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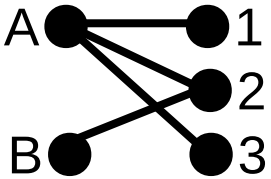

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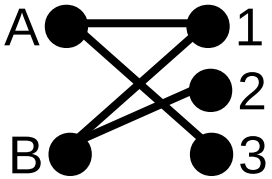

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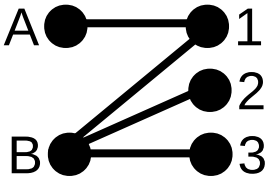

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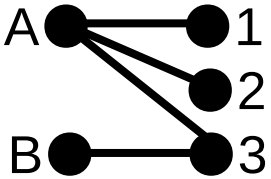

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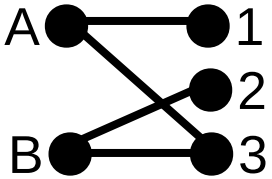

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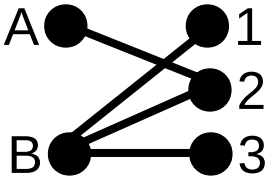

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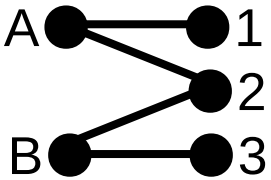

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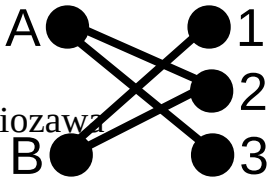

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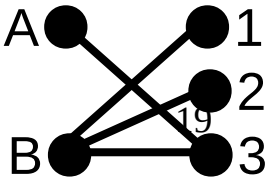

Properties of spanning trees and value determination

- (M,N) spanning tree has M+N-1 edges.
- Contains leaves (vertex with degree 1)
- Start by any value from a vertex of a leaf w_i if country vertex i and p_j if product vertex $_j$.
- Continue fixing the value of a new vertex by eq. $w_i\, a_{ij} = p_j$ when $(i,j) \in T$.
- All vertices are covered (spanning) and no contradiction (no cycle)

Economic meaning of admissible values

- **v** is admissible $\Leftrightarrow w_i a_{ij} \geqq p_j \forall i,j$
  - no production process with excess profit
- If **v** is not admissible, there exists a process with excess profit.
  - If someone comes to know this, new entry occurs and wage-price system will be distroyed.
  - In this sense non-admissible value is not stable.

A in a general position

- Nondegenerate or in general position
- $\mathcal{E} = \{A, \mathbf{q}\}$ as a cephoid.
  - Pallaschke and Rosenmüller 2006, Definition 1.5).
- A new definition:
  - A is in a general position $\Leftrightarrow$

    $T = \{(i,j) \mid w_i\, a_{ij} = p_j\}$ is acyclic $\forall$w, p.
  - We may restrict the range of definition to admissible values.

References:

- Ardila, F., and M. Evelin 2007 Tropical Hyperplane Arrangements and Oriented Matroids, http://arxiv.org/pdf/0706.2920.pdf Now *Mathematische Zeitschrift* **262**(4) : 795-816. (2009)
- Pallaschke and Rosenmüller 2006 Cephoid: Minkowski sum of prisms, *J. Glob. Optim.* **35**: 321-341.
- Ricardo, D. 1817 *The Principles of Political Economy and Taxation.*

Thank you.

- Questions and comments welcome.