On Ricardo's Two Rectification Problems

Y. Shiozawa

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§ 1. Introduction

This paper examines two rectification problems, both related to Ricardo's value theory. One is related to domestic (or internal) value theory and the other to international value theory. First problem is discussed form sections 3 to 6 and second problem from sections 7 to 10. Section 2 argues why Ricardo is still important, even crucial to economics. Final section concludes.

Any rectification problem is composed of two sides: one is to discover the correct solution and the other is to recognize various elements which have been obstruction to the correct solution. Normally two sides come clear almost simultaneously, for one gives light to another. But in some cases, one side comes much earlier than the other.

In the case of domestic value theory, the right solution was presented by Sraffa (1960), but the obstruction side has not been made clear, partly because Sraffa did not provided

suitable explanation and partly because researchers who are interested in classical economics thought to save classical economics altogether. It is necessary to recognize that classical economics are full of contradictions. Consequently, the screening is important. The task is to screen or discern which are to be conserved and which are to be abandoned. In case of domestic value theory, the obstruction side is now more important than the correct formulation side. In comparison to neoclassical economics, classical economics has a wider view on the economy. It was a good part of classical economics but it made many kinds of garbage accumulated. In order that classical economics will be resurrected as the 21st century economics, recycling work is urgently requested.

International value problem trekked much more sinuous path. Ricardo could not indicate even a hint to a possible solution. What he showed was the existence of the problem. He showed most impressively that the value theory in one country cannot be applied to international trade situation. Comparative advantage was an example how the international transactions are different from internal transactions. When young John Stuart Mill attacked the so called indeterminate terms of trade problem, he had no intention to make an important turn of direction which finally led to the neoclassical revolution in 1870's. This paper first refers to a new result in the international value theory and argues why Mill's "solution" was doomed to open the way to neoclassical revolution.

When I speak of "neoclassical revolution," it is the great turn of economics which took place in 1870's. It is usually called Marginal Revolution. I prefer to call this turn of economics neoclassical revolution, because the standard denomination puts two much emphasis on the introduction of marginal analysis, or mathematical formulation in general. It is one of many aspects of the turn, but the most important change from classical economics to neoclassical economics was, as J. R. Hicks pointed it, the change of viewpoint form production to exchange. In other words, it was the change from Plutology to Catallactics. I will argue that Mill's solution was a crucial turning point from Plutology to Catallactics.

§ 2. Ricardo in the 21st century

Why do we concentrate our attention on Ricardo nearly two hundred years after the publication of his magnum opus? Knight (1935) once told that his assumption was that

"ancients" in such a field as economics was to learn from mistakes. If his assumption is correct, what we should learn from Ricardo is to find where and how he committed errors. I stand on an assumption totally opposed to Knight's conception on the history of economics.

Knight considers that economics has grown from crude classical economics stage to refiner modern economics stage. In short, economics science approaches closer to truth as it develops. Some historians of science call this view history of science "Whig interpretation of science. According to this interpretation, we can learn from studying classical economics by detecting errors it committed and by knowing how those errors were corrected subsequently. Knight (1935) summarized Ricardo's economics as a set of seven aberrations.

More moderate and more appreciative interpretation of classical economics is to evaluate classical economists as precursors of more modern theory. We can see full of such comments. We read many comments which treat Ricardo's differential rent theory as a forerunner of marginal productivity theory. Morishima (1989) appreciated Ricardo as a special version of Walras type general equilibrium theory.

My interpretation of history of economics is different from Knights, for I support "dual development" interpretation (Hollander, 1979). As Hollander put it, this conception is common both to Schumpeter and to a variety of Marxists where M. Dobb is a typical representative of the latter. Dual development interpretation thinks that there are two strands of economic doctrines which persisted from the time of Adam Smith and is still claiming that they are inconsistent with each other. It is easy to see that Marxian economists hold such an interpretation, for they claim that mainstream economics is flawed and their system is right. It is interesting to note, after Hollander (1979), that Schumpeter (1954) stood on the dual development interpretation. The "ideal type" of economic analysis for Schumpeter was, as it is well known, Walrasian general equilibrium. All history of economics, for him, is a process of a discovery and that of infiltration of this idea. Ricardian economics was only a detour to this right destination. Adam Smith, J.B. Say, Lord Lauderdale and T.R. Malthus had a correct insight and were approaching to a right direction. Schumpeter thought, as Hollander cites, this detour became inevitable because of Ricardo's "inability to deal with systems of simultaneous equations" (Schumpeter, 2001, 543; Hollander, Library, H11).

Even though I hold the same dual development interpretation as Dobb and Schumpeter, my weight is on Ricardo. In my opinion, the general equilibrium framework is the main cause of the present-day disastrous state of economics and remains the main epistemological obstruction to a reconstruction and further developments of economic science.

Ricardo was the only person who saw clearly the opposition between the classical value theory and the perpetually popular demand and supply equilibrium theory. To reconstruct a new system of economics, it is now necessary to revive classical value theory. This is the reason why I concentrate my efforts in modern reconstruction Ricardian theory.

§ 3. The theoretical core of Ricardo's economics

There are many important figures among classical economists. What is the difference of Ricardo? Hollander cites Thomas De Quincey's comment on Ricardo's contribution to the economics:

> All other writers had been crushed and overlaid by the enormous weight of facts and documents; Mr. Ricardo alone had deduced, a priori, from the understanding itself, laws which first gave a ray of light into the unwieldy chaos of materials, and had constructed what had been but a collection of tentative discussions into a science of regular proportions now first standing on an eternal basis. (Hollander, Library H3)

What discriminates Ricardo from other great philosophers of classical economics lies in the fact that Ricardo alone succeeded to build up a logical system on which to process his economic reasoning. Other great economists knew economy as natural historians probably better than Ricardo but their knowledge was a set of correct (and wrong) information on the economic state of the past and the present. It was not organized as a system. In Ricardo, economics became a science i.e. systems of concepts and hypothesis, organized into theoretical knowledge such that results of logical reasoning can be confronted with experience.

In a loose sense, both Ricardian system and neoclassical economics are sciences. Construction of those two economics is quite different. They form two different traditions in economics. They are two different paradigms. No simple criterion exists by

which to tell which is better. Popperian tests by falsification do not work here, for the very basis of the science is questioned. Simple tests or falsification does not make sense. Both systems contain many unsatisfactory elements but they are also developing and improving. Dual development interpretation is not a simple observed fact. There are two competing paradigms. Contrary to Schumpeter's judgment, it is not at all evident that neoclassical economics or Walrasian paradigm proved to be victorious. Fight continues. The stake is big. It is the future of economics.

Although I believe that the final truth lies in the side of Ricardo, it does not mean that what Ricardo thought and logics he used are all right without exception. Possibility of detecting logical errors and inconsistency with history is a sign that a system is a science. Science develops with the recognitions of logical flaws, and particularly by an effort to construct a new theory in which logical and factual consistency is gained again. There are many auxiliary elements which should be discarded in the original Ricardian system.

As it has been pointed out by many, Ricardo committed many errors. Then, it is important to distinguish which are the bases of reconstruction and which are the irrelevant parts to the core and discarded voluntarily. In this regard, it will be helpful to distinguish Classical Economics and Classical Value theory. Value theory is the core of Ricardo's economics and by consequence the core of Classical Economics. But Classical Economics contains so many conjectures and suppositions which turned out to be wrong as bases of any economic predictions: Malthusian population theory, subsistence wage theory, wage fund theory, decreasing returns to scale assumptions[1], quantity theory of money, and so on. I do not deny that Ricardo himself employed these hypotheses in a place or other and advanced his analysis on the bases of some of these. Despite the Malthus's gloomy prospect, world population increased enormously (perhaps excessively) and their standard of living improved for most of them. In this sense, Ricardo's worry on the possible long run stagnation was baseless. However, this does not mean that Ricardian value theory was erroneous. The very core of value theory is independent of many conjectures that classical economist made on their days and future.

[1] Ricardo cared too much about decreasing returns to scale but did not ponder on the tremendous importance of increasing returns to scale. Classical economists posed wrong weights with respect to returns to scale.

When we reduce Ricardo to the minimal theoretical core, we see also problems he left after him. There are at least two big problems. One is the correct formulation of his value theory. The other is to construct a correct theory of international values.

§ 4. Cost of production theory of values

There are also many misunderstandings of Ricardo's value theory. A typical error is the understanding that Ricardo held labor theory of value. As he mentions at various occasions, he knew very well that the profit should be included in the cost of production (Ricardo, 1951, p.47 note, p.73, n.; Library, I. n.7.; II. n.9.). If we use a less harmful name, Ricardo's value theory should be called "cost of production theory of value."

Most important question is what the cost of production is. According to Senior(1850, Library 4.36) the term "cost of production" was originally introduced by Ricardo himself but the content of the cost of production at the time of Senior was still ambiguous and without misconceptions. Senior understood that the Ricardo's cost of production was "the quantity of labour which has been bestowed on the production of a commodity." Torrens considered that the cost of production was "the amount of capital expended on production." Senior's own definition was "the sum of the labour and abstinence necessary to production."(Senior, 1950, Library 4.45) John Stuart Mill also interpreted that Ricardo's cost of production was equal to the "quantity of labour required to produce the article." (Mill, 1844, Library IV..19) Most conspicuous fact with these definitions is that the cost is considered as a kind of sacrifice which was necessary to obtain a product, and not as an amount of value. By the time of Senior and J.S. Mill cost of production conserves a meaning which can be called "real cost." The change of the concept was necessary. But in front of it, there was a deep cleft

Schumpeter (1954) committed two errors concerning Ricardo. As mentioned in Section 1, Schumpeter blamed Ricardo's "inability to deal with systems of simultaneous equations." But, his assessment is totally ahistorical. Although modern studies of simultaneous equations started in 17th century (in Europe and in Japan), general theory came to be known in 19th century and linear algebra was introduced in the university teaching only in the first half of 20th century[2]. It is quite natural that Ricardo and other

[2] Even though he opposes Cayley-as-Founder view, Hawkins (1974) thus summarizes: "Although the origins of the theory of matrices can be traced back to the 18th century and although it was not until the 20th century that it has become sufficiently absorbed into the mathematical mainstream to warrant extensive treatment in textbooks and monographs, it was truly a creation of the 19th

classical economists was not versed in systems of simultaneous equations. Schumpeter's favorite Walras discussed simultaneous equations but he remained to count number of unknowns and number of equations. Existence of positive solutions came to be examined only in 1930's in Vienna (Weintraub, 1984, Part 2).

Second error Schumpeter (1954) committed lies in his belief that every simultaneous system should take a form of demand and supply equations. In this point Morishima understood better than Schumpeter. He thought Ricardian system can be reformulated into a system of equations, but not that of demand and supply equations. Morishima was too absorbed in general equilibrium framework and could not discern the central opposition between classical and neoclassical economics but he was free from Schumpeter's second error.

A coherent definition of cost of production requires a system of simultaneous equations, for the value of goods which enter as inputs must be counted as cost of productions. This work has been done by Sraffa (1960). A crucial difference between Sraffa's system and Walras's and other general equilibrium system is that Sraffa's system is composed of only price equations with a set of inequalities with regards to quantities (Sraffa, 1960, Ch. 2). In the neoclassical tradition, it is the equality of demand and supply in each good that determines its price. In Sraffa's system prices are determined independently from demand and supply. To be more precise, the system contains as variables prices, wage rate and profit rate. The value variables are determined by themselves. This is the core of cost of production theory of value. Demand and supply are but disturbing factors of prices.

In the time from Smith to John Mill the capitalism was still young and it was the middle merchants who determined market prices by the movement of demand and supply. Market prices fluctuated everyday but classical economists had an insight to see long run constant prices around which market prices gravitate. But the capitalism evolves. At the turn of century from 19th to 20th, producers became more or less monopolistic and started to set prices for their products. This pricing behavior came to be known in 1930's as full cost pricing or mark-up pricing (Oxford Economic Surveys).

The new pricing behavior was more reasonable for producers or suppliers. Prices became more stable and it became easier for producers to plan and realize cost downs.

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century."

The gravity center of profit source shifted from middle merchant type windfall profits to producers' profits which come from cost downs and new products. The new pricing behavior required firms to change their producing behaviors. In order to keep product price stable, it was necessary that producers adjust their production volume more closely to demand flows.

§ 5. Full cost principle or redefining the cost of production

It is well known that Sraffa started to write his book in late 1920's. It is not a mere coincidence that Sraffa (1926) concluded that it was not the rise of marginal cost but the inability to expand sales volume when they want to expand their activities. Sraffa clearly separated two regulation problems: pricing and quantities.

Sraffa solved Ricardo's first problem by discovering how to define cost of production and re-establish the cost of production theory of value. It will be better to add some more words on Sraffa (1960). We should not think that the task of developing classical value theory and that Sraffa understood completely what was put at the stake. He was very loyal to last day Ricardo and tried to find a kind of absolute measure of values. He proposed use standard commodity as such a measure. In my opinion, it is a useful tool to see tradeoff relations between wage and profit rates. But we cannot use it when technology changes. In this sense it is far from invariable standard on which Ricardo imagined to base his analysis. There is no such invariable standard and we should search other methods. Sraffa (1960) emphasized that his theory dose not assume constant returns to scale. This remark was useful to show that value theory does not require marginal change of production factors. But if we confine ourselves to fixed production volumes, we cannot go further. Constant returns to scale are the strong case (after Ricardo) for classical value theory. From constant returns to scale we should go to the increasing returns. This is the most disputed point between classical economics and neoclassical economics. Standard equilibrium theory cannot incorporate increasing returns, for it is impossible to define the supply function in that case. Reconstructed classical economics requires no such function. Production is regulated by the actual flow of commodities. Short run adjustment occurs in quantities and not in prices.

It is convenient to mention two more points no less important. Full cost principle made it possible to put an end to the dichotomy of value theory, i.e. ever fluctuating market price, determined by demand and supply, and virtual long run exchange value,

determined by cost of production. In the time of classical economists, it was not clear that producers fix prices and demanders (both industrial procurers and consumers) determine quantities to buy at the presented prices. If we keep dichotomy between long run and short run prices, value equations are rather virtual ones and do not regulate real exchange ratios. They indicate exchange ratios only in average in the long run. When the pricing and quantity adjustment behavior change, the value can play more direct role than it was one step mediated average regulators. For example, we can define effective demand much more correctly than Keynes did with his peculiar aggregate demand and supply functions. Effective demand is in fact what limits production volume of each firm. Sraffa (1926), Keynes (1936) and Sraffa (1960) can be combined to a unified theory.

§ 6. Classical value theory vs. neoclassical value theory

I have to be brief in this comparison. Two different paradigms are at the stake. Many discussions are already made and I myself have discussed this question in various occasions[3]. I confine myself to rectify some misunderstandings concerning the demand theory of classical value theory.

Repeated allege against classical theory was that it lacks demand theory. Jevons and other neoclassical economists claimed that they have incorporated demand theory (by way of marginal utility theory) and redressed partial character of classical theory which was based on supply side. This claim contains a half truth. The demand theory is still weak part in comparison to production part. But this claim is only a half truth. The theory of demand they provided is full of deficiencies. If we calculate the necessary computing time to find one's set of commodities which gives maximal utility, it comes out that computing time exceeds easily with some tens of commodities and the neoclassical program to define demand function is practically impossible (Shiozawa, 2003?). If the income distribution is not of special type, Sonnenschein-Mantel-Debreu theorem tells that excess demand function can be arbitrarily close to any continuous functions provided that it satisfies Walras law (Keen, 200X, ?). In this sense, neoclassical demand theory is full of deficiencies despite of highly mathematical appearance. The most important fact is that neoclassical general equilibrium framework denies the necessity of sales efforts including marketing. Neoclassical

[3] See for example Beinbocker (2006: 2007), Keen (2011) to cite only two recent appraisals. See also Shiozawa (2004).

equilibrium economics with its demand theory is constructed on the fictive economic reality.

In the back of the half truth, there is a half lie. It is a complete lie that classical economics neglects demand. Classical economics has a demand theory different from neoclassical economics. If the prices are fixed, demand does change in classical framework, for it determines production volume and then labor and input demand. In the medium run, the average level of production and the growth rate are the crucial factors of new investment. The main task of analysis is to make clear how the whole process goes on. Separation of value determination and quantity determination is the crucial aspect of classical economics. Prices are determined primarily independent of quantity side. This framework is not only closer to reality but also affords a new research program in the line that Keynes once imagined by the notion of the principle of effective demand. Sraffa (1960), interpreted in this wide perspective, provides what Ricardo struggled to reach with successive reformulation of his value theory.

The first rectification problem of Ricardian value theory was thus finally solved. From Ricardo to Sraffa, it took more than 140 years. A lesson of this story is the necessity to recognize that theory evolves. Theory develops only through introduction of new interpretations and theory constructions. Another lesson is that there are two competing paradigms and it is vital to discern essential difference of two paradigms.

§ 7. International value problem

The second big problem left after Ricardo was the construction of an international value theory. Ricardo knew that his value theory was not applicable to international trade situation. He expressed this point by the following comment: “the same rule which regulates the relative value of commodities in one country, does not regulate the relative value of commodities exchanged between two or more countries” (Ricardo, 1951, p. 133; Library 7.10).

Evidently it is necessary to construct a new theory, a new international value theory. If the correct formulation of his value theory was vital to his whole system, the task of constructing a new theory of international values was as important as the first problem, for the foreign trade became more and more important as the time passes by. If there were no good formulation for Ricardo’s value theory, his whole system would crash down.

If there were no international value theory, Ricardo's theory is a very restricted theory which is only applicable to transactions within a country.

In this sense, task of constructing an international value theory was as vital as the first problem, that is, the correct formulation of cost of production theory of value. Despite of this crucial importance to Ricardian system, this question was not posed correctly nor was discussed as such. We may indicate many points why this important question was left unnoticed. It is true Ricardo did not try to present his tentative theory in the case of foreign trade. It is possible that many readers of Ricardo thought that it was too difficult to try to construct such a theory. More probable story is that readers misunderstood that the comparative advantage theory was all that was requested.

Comparative advantage illustrated by his four magic numbers is still the most popular topic in undergraduate textbooks of international economics. Researchers in Ricardian tradition still discuss specialization patterns before inquiring what values hold in the world. As M. Bowley (1937) observed, there are two traditions in the theories of international trade. One approach tries to obtain specialization patterns by means of "real" costs or physical costs, i.e. labour and material input coefficients. The other approach is to start from value relations. It is true that the first real cost analysis has been a long tradition since Ricardo and John S. Mill. Even now specialization pattern is investigated mainly by the first approach. In 1930's, Ohlin claimed that the second approach was the only hopeful way to develop a more realistic theory of international trade. Ohlin's new approach opened the way to a complicated distortion.

As it is well known, Ohhin (1933) presented a new strand of trade theory. It is now called Hechscher-Ohlin theory or Heckscher-Ohlin-Samueslon theory adding the name of the modern style interpreter (HOS theory hereafter). Some researchers even in the non-mainstream tradition understand that HOS theory is a natural generalization of Ricardian theory of international trade to the case with more than two production factors[4]. University level textbooks explain that HOS theory is a modern form of Ricardian theory. Ricardian system excluded material inputs such as parts, materials and machines. HOS theory treats capital explicitly and thus supplements the deficiency of the Ricardian trade theory. This understanding requires the first rectification.

HOS theory is constructed in the neoclassical framework in all aspects. It starts from

[4] See for example Boyer and others (2012, p.332) and Rowthorn (2013, p.13)

given factor endowments. It assumes full utilization (hence full employment) of all resources. Price plays the main adjustment function. It is organized after the principle of demand and supply. It assumes smooth substitutability for production inputs. One of a few discrepancies between HOS theory and the standard neoclassical theory is that HOS theory does not assume personal maximizing agents explicitly.

In Ohlin's understanding he made a revolution in trade theory, for he succeeded to change real or physical cost tradition into value relation analysis. In this point, Ohlin was right. The first method of analysis has been hitting the wall. Various trials from Marshall, Taussig, to Viner could not present a good result for the case of many countries and many commodities. There was a fierce debate between Viner's real cost approach and Harberler's opportunity cost approach. Even after Ohlin (1933), many other trials continued. Following McKenzie's contributions, Jones (1961) succeeded to give a formula by which to know which type of specialization is possible for the case with the same number of countries and commodities. Ethier (1999, p.764) praised Jones's achievement in this expression: "[Jones's] contribution was so definitive that the Ricardian model has since been used almost entirely as a tool for other purposes and not as a subject of research in its own right." Ethier was totally mistaken, for the task of constructing a new theory of international values was left unsolved. The necessity of a new theory was not even recognized at all. It was necessary to seek a new value theory and in this unique point Ohlin was right.

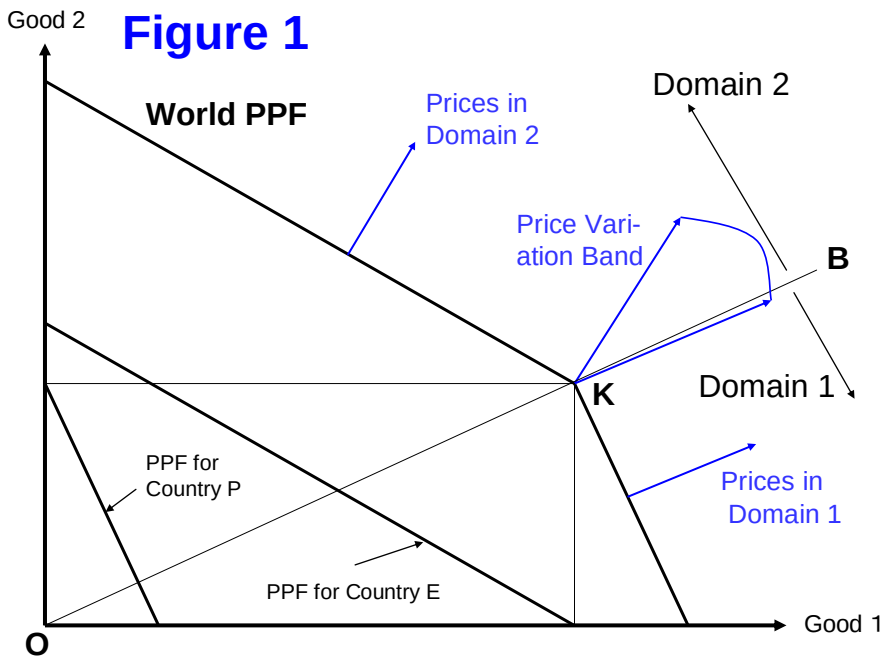
After Ricardo recognized the necessity of constructing a new theory of international values, nearly two hundred years have passed. It was only recently that a new theory was obtained for a sufficiently general setting. The reason of this long delay can be explained partly by the Ohlin's thesis. Trade theory should be constructed as a value theory and it should not be developed by the physical cost approach. Taking this factor in consideration, two hundred years are too long and it is necessary to give light to an accident which took place just 10 to 15 years after Ricardo's first exposition of his trade theory. That is the meaning of John Stuart Mill's "solution".

§ 8. Mill's misdirected "solution"

I don't doubt John Stuart Mill's goodwill when he believed that he had advanced Ricardo's theory one step forward by his solution. Ricardo's example was the situation where two products are exchanged between two countries. It is evident from the latter

half of his famous chapter on foreign trade that Ricardo thought there were many commodities besides cloth and wine. It is now also known that Ricardo thought that the exchange ratio (in international trade) between cloth and wine is determined (Yukizawa, 1974; Maneschi, 2004). Ricardo's system contained three set of prices, i.e. two systems of internal or domestic values and a system of international values. This is only conceivable when the volumes of international trade are small and their influence to domestic values is minimal. Mill was right to think that this is not the final situation when trade continues and volumes increase. He wanted to determine what value system (a single value system) would prevails in the long run. The problem he set was to determine the exchange ratio between cloth and wine, or exchange ratio between exported items and imported items. The latter is customarily called terms of trade.

Mill thought that the terms of trade were not determined in Ricardo's exposition and searched possible logic to determine them uniquely. In addition, Mill wanted that both trading countries enjoy gains from trade. In two-country, two-commodity situation, Mill was obliged to observe a very special case as an economy of international trade, i.e. the case of complete specialization. Complete specialization means that a country has only one commodity which is competitive in the world market. In this situation, when the labor input coefficients are fixed and each country has a determined quantity of labor (or labor power), the quantity of the product a country can produce competitively is determined. In Mill's example (Mill, 1848, Library III.18.6-17.), Germany specializes in linen and England specializes in broadcloth. If a two country two commodity model is interpreted strictly to terms, then Germany employs all workers to produce linen and England employs all workers to produce broadcloth. As the labor power is determined, the quantity of linen Germany produces and the quantity of broadcloth England produces are determined.



If we simplify the situation, Germany holds X quantity of linen and England holds Y quantity of broadcloth and both sides want to make a transaction in such a way that each country maximizes their satisfaction. This is nothing but a pure exchange economy. Mill made this setting and tried to determine the exchange ratio between two parties. Ricardo considered a production economy and enquired what kind of international values will prevail. J. S. Mill shifted effectively the problem into an exchange economy. The concrete method Mill employed in determining terms of trade is not important. It is ordinarily explained that Mill analyzed this situation by reciprocal demands. But it is only a superficial understanding. Much more important aspect of Mill's "solution" was that he transformed world production problem into an exchange problem where countries are provided with predetermined amount of commodities.

The situation can be easily illustrated by Figure 1. There are two goods: good 1 and good 2. Two counties: country P and country E trade with each other. Figure 1 expresses a situation of a standard two-country, two-commodity model. PPF is the abbreviation of Production Possibility Frontier. It is the set of maximal points of the PPS or Production Possibility Set. When the population changes, the triangle enclosed by two axes and PPF become greater or smaller but the slope remains the same as far as labor input coefficients do not change. World production possibility set is given as the sum of two

vectors, each belong to the production possibility set of each country (Minkowski sum of each country's PPS). World PPF keeps its form as kinked line with two line segments, however the constants of the model changes (the constants include each country's labor power and labor input coefficients). The unique kinked point is named K. The world PPF is divided into two parts: Domain 1 and Domain 2. At any point in each of these domains (or line segments in this case), the prices are proportional to the prices of a country's closed economy. In the case of Figure 1, the prices are proportional to the prices of Country P in Domain 1 and to the prices of Country E in Domain 2. Thus, any situation either in Domains 1 or 2 is excluded from Mill's examination, for one of two countries has no gains from trade in such a point of the world PPF. The only possible candidate to be examined is the point K. At this point the prices have some margin of variations which we call price variation band.

J. S. Mill examined this situation and explained that the terms of trade would be determined in such a way that the export and the import balance. In this analysis Mill imagined what will later be called reciprocal demand curve[5]. The prices or terms of trade are determined as a cross point of two demand curves. As a demand curve of a country is a supply curve of the other country, this is in essence equivalent to demand and supply curve analysis. Mill questioned the existence of this balance point and argued uniqueness of the equilibrium. In this point Mill is mathematically speaking very modern. Most of classical and some early neoclassical economists did not bother with the existence of a solution[6].

Mill's solution gave a heavy impact on the fate of economics. This can be traced in two aspects. The first consequence was the logical framework of Mill's own economics. He believed to be a Ricardo's loyal disciple. Mill's main emphasis in his *Principles of Political Economy* (1847) was Ricardo's cost of production theory of value. But his "solution" of his trade problem obliged him to abandon this core of classical value theory. In his *Principles*, Mill writes as follows:

> Since cost of production here fails us, we must revert to a law of value anterior to cost of production, and more fundamental, the law of demand and supply. The law is, that the demand for a commodity varies with its value, and that the value

[5] It is pointed that the phrase "reciprocal demand" was coined by Torrens (Viner 1937, p. 536; Humphrey 1995, p.42)

[6] In the next section, we will examine the cases of Jevons, Marshall and Edgeworth.

> adjusts itself so that the demand shall be equal to the supply. This supplies the principle of repartition which we are in quest of. (Mill, 1847, Library III.16.5)

This passage appears in Book III, Chapter 16, where Mill discusses "Some peculiar cases of value." His peculiar cases here mean commodities of joint cost of production. Mill's example is joint production of coke and coal-gas. They are produced by the same process from the same material. But these are rather rare (hence peculiar) cases[7]. One may doubt if these exceptions really deserves the change of principles. I suspect that Mill wanted to attenuate the readers' shock. It is possible that Mill prepared the readers to the next two chapters in which he treat international trade and value. His international value theory depends on the "anterior law," that is, law of demand and supply. But the case of international trade is so important and universal that one cannot think this is an exception. An evidence to support my guess is that Mill wrote a similar phrase in his earlier writing as follows:

> The principle, that value is proportional to cost of production, being consequently inapplicable, we must revert to a principle anterior to that of cost of production, and from which this last flows as a consequence,—namely, the principle of demand and supply. (Mill, 1844, Library I.19)

The title of the writing was "Of the Laws of Interchange between Nations" and published as the first essay of 5 of them in Mill (1844). It was written around 1829 to 30, but it is known that Mill worked on the problem when he was still junior. This was the starting point of young Mill's independent research work. The conclusion he extracted from this research inevitably played a big role in the making of his system of economics but gave a much more great impact to the course of economics after Mill. In the next section, we examine the cases of three representative economists who lead the neoclassical revolution in economics.

§ 8. The Origin of Neoclassical Revolution / An Internalist View

I have not explained how Mill's "solution" was misdirected. It is the task of the next section. Before proceeding to this task, let us see the consequences of Mill's "solution." My bold conjecture is this was the very point of return from classical economics to

[7] Joint production which arises by treating fixed machines and installations as by-products causes no problem, for they normally remain within a firm. Values of older machines and installations are defined by a system of equations. See for example Sraffa (1960) Chap. 7.

neoclassical economics, or in other expression conversion from Plutology (economics of production) to Catallactics (economics of exchange). As this is a conjecture which came to my mind recently, and as the question is extraordinary big, I do not claim that this section provides sufficient evidences to prove my conjecture. It requires enormous work in the history of economics and I have to ask someone to try to verify or falsify my conjecture. This section remains a rough description of what may have possibly occurred.

John Stuart Mill did not abandon the very core of classical value theory but he was obliged to make a structural reform of the logic of value theory. He adopted, if reluctantly, the law of demand and supply as anterior and more fundamental law than the cost of production theory of value. Ricardo once declared that "[t]he opinion that the price of commodities depends solely on the proportion of supply to demand, or demand to supply, has become almost an axiom in political economy, and has been the source of much error in that science." (Sraffa p.382; Library 30.3) Mill knew this and had to reconcile Ricardo's value theory and his new "solution". What Mill did was to admit the law of demand and supply as more general and more fundamental law and situate cost of production theory as something applicable to narrower situation. Mill stopped at this point, for this was the maximal possible concession he could make. But economists after Mill did not. Here comes the second consequence of the Mill's "solution".

If the law of demand and supply is more fundamental, it is natural to apply this law uniformly and universally. My suspicion is that English founders of neoclassical economics were influenced by the Mill's "solution" and his conclusion. Jevons, Marshall, and Edgeworth show more or less indicative evidences for this suspicion. As Jevons is thought to be the person to have leaded the marginalist revolution, examination of Jevons will inevitably be longer than Marshall and Edgeworth.

I do not deny that other factors intervened in the arrival of the neoclassical economics. Spread of optimization techniques (in mathematics), tradition of utilitarianism (in philosophy), deep-rooted tradition of demand and supply (in economics) and finally the general fallback of Ricardian economics. Mirowsky (1989) argued that "energy" concept was crucial to the arrival of neoclassical economics. All these factors worked indeed. But we should ask how direct these factors worked in the formation of exchange economics in place of production economics. Their influences are indirect, whereas impact of Mill's "solution" was, I believe, more direct than others. As far as I know, no one has ever

claimed that Mill's "solution," or his situation setting when he wanted to determine know how gains from trade were divided between trading countries, was one of key factors on the arrival of neoclassical economics. I believe this conjecture deserves a scrutiny.

8.1 William Stanley Jevons

In the case of Jevons, my judgment is symptomatological. Despite his apparent hostility against Mill, and although there are no direct evidences, it is highly probable that Jevons unconsciously accepted Mill's fundamental framework but for one or two core ideas. It is possible that Jevons started to come up to his theory of exchange with this framework. Of course, his primal motive was to apply utility maximization to explain economic affairs, but it seems that Jevons had in mind Mill's theory on international values.

First, in his Theory of Exchange (Jevons, 1871, Chap. 4), Jevons uses a terminology which evokes international trade. Jevons starts his argument by supposing two "trading bodies". As far as I know., this is a peculiar term was not employed in academic works before Jevons. Sidgwick (1901) and Marshall (1920) used this term, but only when they mentioned Jevons's work. Jevons explains that this term can stand for individuals, and for whole population of a continent. In his phrase, "[t]he trading body may be a single individual in one case; it may be the whole inhabitants of a continent in another; it may be the individuals of a trade diffused through a country in a third." Gains by exchange, the term used as the title of a section of Chapter 4 of his *Theory* (Jevons, 1871, Library IV.32) is also evocative of now ordinary expression "gains from trade". John S. Mill used terms like "gains by foreign trade," advantage or/from the trade,".and "advantage of (foreign) commerce" in Mill (1847) and "gains of commerce" in Mill (1844).

Choice of terminology is important, even crucial some times. Jevons knew this very well, for he discussed scientific methodology in many occasions. In the case of "gains by exchange," Jevons explicitly mentions Mill and argues the concept. The change Jevons added was to extend this concept from the gain from foreign trade to all situations of exchange. If this is to introduce a concept similar to consumer surplus, he had a good reason. The question is why Jevons adopted the term like "trading body" in order to express fictitious subject of exchange. He should have reasoned that there is perfect similarity between two person exchange negotiation and the international trade situations. There is a great ambiguity and this identification may cause conceptual

confusions, as it was criticized later by various economists. Why did Jevons adopt the term "trading body" at the risk of all possible confusions? My conjecture is he framed his theory of exchange on the basis of Mill's "solution" on international trade. Mill succeeded elegantly to analyze how the terms of trade are determined. Jevons's ambition may induce him to think of the necessity to go beyond Mill in this point. With his theory of exchange, Jevons may have believed he had succeeded in generalizing Mill's method of analysis to all exchange situations.

The second observation concerns the relationship between Jevons and Mill expressed in the text of *The Theory*. In the Theory of Exchange chapter, Jevons mentions Mill many places (8 paragraphs) and complains Mill's misconceptions. His first complaint may be well founded. He mentions two times in Chapter IV Mill's contention that theory of value is complete (IV. 2 and IV.51). This is must be Mill's slip of pen, for no one can claim that a theory is complete and perfect. There are always some spaces of development. The question is whether they are still large or not. Jevons's second complaint is that Mill did not firmly declare that value was not a substance but a ratio which express exchange relations. Jevons argues this point in two places again (IV.4 and IV.38)[8]. He even cites Mill's text in the latter:

> The idea of a ratio, as between demand and supply, is out of place, and has no concern in the matter: the proper mathematical analogy is that of an equation. Demand and supply, the quantity demanded and the quantity supplied, will be made equal. (Mill, 1847, Library III.2.14)

Jevons cites this text out of context. What Mill wanted to say here by the expression "the idea of ratio is out of place" is not related to a ratio of exchanged goods. Mill wanted to say that the ratio of demand and supply does not determine the upward (or downward) moving rate of price[9]. Jevons is completely mistaken in the interpretation of Mill's text. It is possible that Jevons cited this part after he had searched a good target but could find no better text. Jevons's third complaint is difficult to judge. He claims that Mill's concept of value still conserves the notion that value represents and expresses a substance, although Mill declares that "[v]alue is a relative term" (IV.10).

Other three mentions are related to gains by exchange and Jevons explicitly argues Mill's chapters on international trade and value. Jevons criticizes Mill's concept of gains from trade and presents a new concept which is similar to consumer surplus concept.

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[8] Jevons also mentions this point in the Preface to the second edition.

[9] J.S. Mill is explicit in proclaiming that value is a relative value (Mill, 1848, Library III.6.2).

Despite of these various complaints, and despite of abusive phrases he threw upon Ricardo and Mill, like "wrong-headed man" and "wrong-headed admirer" in Preface to the second edition, Jevons praises two chapters on international trade and values saying that "That theory is always ingenious, and as it seems to me, nearly always true" (IV.100). In the Preface to the second edition, Jevons plays to be surprised by the fact that Mill used mathematical symbols in the two international chapters and add this appraisal:

> [T]he result [of using mathematics] is that these chapters, however tedious and difficult, will probably found the truest and most enduring parts of the whole treatise. (PS.14. [ ] is added by Shiozawa.)

Jevons complains all kind of Mill's misconceptions, but there is a notable exception. Why did Jevons complain of the cost of production theory of value which must be the main target of his criticism? Jevons does not mention this question in Chapter 4. Moreover, Jevons admits that cost of production finally determines the value (in his revision at the second edition after Inoue (2005, p.26)). Later economists, who praised utility theory of value, criticized this off course. Jevons talks of "cost of production" only incidentally in chapter 4 except in the last heading where he agrees that cost of production determines the value. Cost of production is discussed as a subject matter only in Chapter 5 where he discusses labor and production. Even, here, he agrees that the value is proportional to cost of production (V.28 andV.38). What Jevons added in this part is the notion of productivity. He claims that the ratio of productiveness cost is "the reciprocal of the ratio of the costs of productions." This remark is a truism, for this is a definition of productivity. We may talk about various factors which might influence the cost of production. Productivity can be measured only by the cost and measuring cost presupposes the value relations. Jevons only mystified the relations between value, cost of production and his favorite final utility by inserting the concept of productivity.

Plainly speaking, Jevons admits cost of production theory of value. He did not understand what his own contribution was. I do not deny his contribution but it is possible that Jevons could not locate it precisely. This point will be discussed later in the third observation. What we should consider of is why Jevons has accepted cost of production theory of value. He pretends to give a "true theory of value" (III.11) and judges that "[t]he theory of value, as expounded by Mill, fails to reach the root of the matter" (III.38) In spite of these pretentions, Jevons accepts the core of the classical theory of value, i.e. the cost of production theory of value. What Jevons could criticize

was the concept of value. Value is not a substance but a relation. This is an important recognition, for this substance theory of value continued to work as hindrances to the development of value theory, for example, in Marxist economics. Other than this, Jevons's contribution is minor with respect to the causes or factors which determine value short or long run.

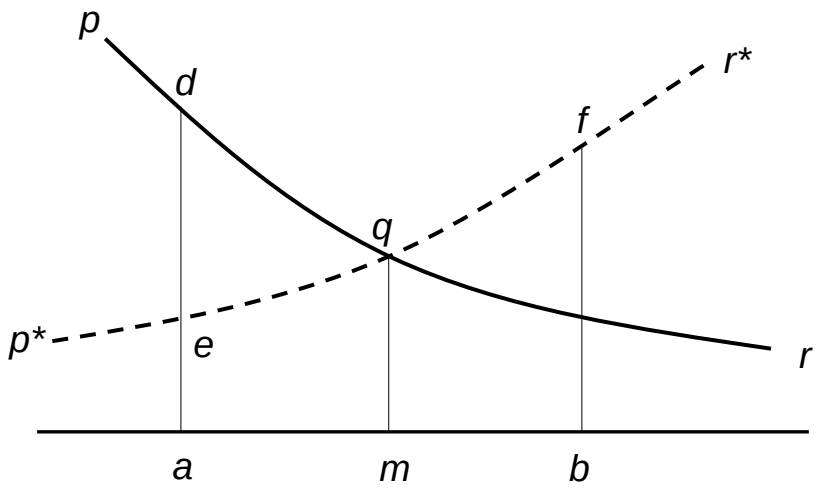
As I will discuss later, Jevons formulated a new theory of demand, but he confused with it the theory of value. In the deep layer of logics, Jevons relies entirely on Mill's theory of value. Jevons made a fundamental change in the logical structure of economics. Classical economics represented by Ricardo and Mill based production as determining factor of whole capitalist economy. Jevons placed utility at core of all economics and presented a theory of exchange. However, Jevons could present no explicit refutation of the classical theory of value or no good reasons why the theory of exchange should precede theory of production. At this point Jevons depends totally on the authority of Mill. Jevons accepts Mill's conclusion that laws of demand and supply is more basic (anterior and general in Mill's words). Jevons criticizes all arguments of J. S. Mill from his science methodology, logics to political economy. But as for the truth of the law of demand and supply, Jevons only cites Mill's conclusion. Mill admitted that in those cases of joint production and international trade, one had to "revert" to the theory of demand and production but he did not claim that the law of demand and supply should be applied to all exchange situations. When he announced his famous "anterior" proposition, Mill preceded it the conditional "Since cost of production fails *here*" (emphasis by Shiozawa). Jevons skipped this part and generalized Mill's concession to all situations. If he wanted to do this adequately, he should refute Ricardo's contention that the costs of production regulates prices or explain how Ricardo-Mill analysis by the cost of production can be better explained by his own theory. Jevons may not be conscious of this necessity in the first edition of the *Theory*. He discusses this question in the Preface to the 2nd Edition but could not arrive to make clear the logical structure of this problem.

Third observation is concerned to the logical structure of Jevons's theory of exchange.
I argue that Jevons did not really understand the logic of cost of production theory of value and that he did not even understand the logic of his own theory of exchange.

Let us start with the examination of Figure V in the chapter IV of the *Theory* (IV.32). This figure is the result of discussions with Jenkins (Inoue, 1987, p.170-173; Maas, 2005,

274-276.). It seems Jevons could not explain this figure persuasively. In later examinations this figure became a cause of confusions. Keynes mistook this figure as a precursor to demand and supply curves (Inoue, 1987, p.173). The figure is reproduced here in a bit simplified version (Figure 2). As introduced in Inoue (1987), Jenkins argued that this figure is valid when one assumes that two goods are exchanged 1 unit against 1 unit. Jevons accepted this comment and explained that two axes of goods are

Figure 2 (Jevons's Figure V, simplified)



"superposed" by taking units of both goods to be of equal length. This reveals the fact that Jevons and Jenkins did not understand the logic of their reasoning. The correct interpretation I assume is to take as units of goods in such a way that both units have the same price or value. This is equivalent to assume an exchange ratio of two goods, but Jevons seems to dislike such an idea. When axes are superposed, the axis of the second good is reversed and a point of the axe represents a state after the exchange of certain amount of goods: one to one in value. If one moves right, the quantity of the first goods increases, whereas the quantity of the second goods decreases. Two curves represent final utility (or marginal utility by present expression). Note that the dotted line represents a negative a value when one moves rightward. The vertical difference between two curves expresses the final utility when one moves rightward. At the coordinate *a,* the final utility of this point is $d - e$ and is positive. One can increase one's total utility if one increases the first goods. The contrary is true when one is situated at the coordinate *b*. In this case, by increasing the second goods, one can

increase one's utility.

What Jevons explains by Figure V is simply a utility maximization when the rate of exchange is given. Jevons himself is confused, for prior to the introduction of Figure V, he speaks of equilibrium of exchange. This diagram does not explain the existence of an equilibrium point or a process of approaching to an equilibrium point. As mentioned above, the maximization is possible whenever an exchange ratio is given. In order to show the existence of an equilibrium state, it is necessary that two person's maximization behavior is consistent.

Although he fails to explain the equilibrium by a diagram, Jevons barely succeeded to do this in the subsequent section. Let the exchange ratio is x to y. Jevons assumes separate utility for goods 1 and 2. When one possesses x quantity of good 1 and y quantity of good 2, then the final utility of possessing goods 1 is ϕ (x) and the final utility of possessing goods 2 is ϕ (y). When we have to distinguish persons who own goods, subscripts 1 and 2 are attached. Suppose before exchange, one possesses the quantity a of goods 1 and the other possesses the quantity b of good 2 and x quantity of good 1 is changed for y quantity of good 2. Then the first person's maximization behavior is described by equation

$$\phi_1(a - x) : \phi_1(y) = x : y. \qquad (8\text{-}1)$$

Likely, the second person's equation is given by

$$\phi_2(x) : \phi_2(b - y) = x : y. \qquad (8\text{-}2)$$

If we assume equations (8-1) and (8-2) a system of simultaneous equations, one may obtain solutions when some conditions are satisfied[10].

Jevons counts number of equations and unknowns and declares:

> The two equations are sufficient to determine the results of exchange; for there are only two unknown quantities concerned, namely, x and y, the quantities given and received. (IV.37)

Jevons knows a possibility that this system does not possess a positive solution (Sections *Failure of the Equations of Exchange* IV.66-77 and Negative and Zero Value, IV.78-88). But, he does not dig down the question of the existence and analyze the behavior of solutions.

Facing a system of equations, mathematicians of 20th century or after are accustomed to

[10] Jevons' actual reasoning is more sinuous, for he makes intervene infinitesimal increments.

examine two problems:

(1) Does a solution exist or not? (Existence problem)

(2) If solutions exist, are they unique or not? (Uniqueness Problem)

As an engineer of 19th century, Jevons satisfies himself by a simple unknowns and equations counting. As I have mentioned already in § 3, the existence of positive solution came to be examined later in 1930's in Wien (Weintraub, 1985, Part 2). I have no intention to blame Jevons but I have to add that Mill's solution is better with this regard as it will be shown in the next section on Marshall.

It will be an interesting question in what way Jevons understands law of demand and supply. Jevons uses the phrase "law(s) of supply and demand" at least 15 times in the main text of the *Theory*. Among these 15 cases, only one relates laws of supply and demand with the price. Precisely it is written that "[t]he ordinary laws of supply and demand treat entirely of quantities of commodity demanded or supplied, and express the manner in which quantities vary in connection with the price" (I.5). It would be difficult judge whether Jevons has a concept of demand and supply functions. There is no expression as "demand function" or "supply function". It is possible that Jevons knew that demand and supply depend on prices but he may not have recognized the importance of demand and supply functions. One may of course construct those functions. It is sufficient to plot demand and supply each time an exchange ratio is given. The point is that Jevons has not come to think of this necessity. In his preface to the second edition, he spoke of demand function when he cites Cournot (PS.20).[11] In the time when the demand and supply functions are so common, it is a bit difficult to imagine how Jevons reasoned without these concepts. Some economists after him criticized that Jevons confused utility function and demand function. This kind of criticism is invalid, for Jevons did not think in terms of demand and supply functions when he spoke of "laws of supply and demand."

Except the Chapter 1 when Jevons talk about laws of supply and demand, there is no discussion how the equilibrium is attained in Chapters IV or V. He speaks of "equilibrium" 10 times in Chapter IV. It is not clear what Jevons meant by the word "equilibrium." Some of them stand for stationary state. Some of them stand for balance of two or more forces or motives. Consistencies of demands and supplies are never mentioned. It seems that Jevons was really content by the equality of unknowns and

[11] If I cite Jevons exactly, he expressed "demand for a commodity is a function of the price." He may not have a firm concept of demand and supply function.

equations.

Now let us return to the main theme of our discussion. At the beginning of his book, Jevons declares that "value depends entirely upon utility" (I.2). At the end of Chapter IV, Jevons admits that cost of production may determine value via final degree of utility (IV.131). How can we reconcile this seemingly contradiction? Jevons explains that labor and thus production, in large proportion of cases, is the determining circumstance. What does this mean? Let us enquire if Jevons's argument is right or not.

Here Jevons comes very close to classical theory of value, i.e. cost of production theory of value. Given a rate of exchange or values, individuals behave so as to maximize their utility, as it is explained above. Utility including final degree of utility operates only in this phase. But there is no warrant that any arbitrary chosen set of values gives consistent demands and supplies. Suppose there are some inconsistencies. What will happen? Neoclassical tradition is to answer that values change. However, this is not the unique possibility. What happens when production changes? If producers can well adjust volumes of their products, demands and supplies will soon become equal. Here lies the key point. Classical economists (including contemporary classical economists) consider that quantity adjustments are quite rapid in most of industries, especially in manufacturing. In the time of Smith and Ricardo, main ingredients are agricultural products. In that case, adjustment takes place one a year. In the meantime it was necessary to redress equality of supply and demand and this necessitates changing prices. But as fast as quantities are adjusted to demands, it is not the relations of demand and supply, for supply is always approximate to demand. This is the typical situation that classical economists consider ordinary. In this case, it is the competition which derives capitalist to keep down their prices as low as possible, but above or equal to cost of production including normal profit. Thus prices become normally equal to full costs.

In the time of Smith and Ricardo, it was always necessary to wait until supply adjusts itself to demand of the market. In the meantime, prices fluctuated. Distinction between prices and values or that of spot prices and long run average of prices were necessary. In 20th century and of course in 21st century, quantity adjustment is predominant for most of commodities. The logic of classical theory of value has become much clear and simple.

If we return to Jevons, he could not recognize what is the determining circumstance. It

is the adjustment of quantity and cost of production. Jevons, clinging to his idea of utility, could not discern the social function which determines values. As he recognized, individuals adjust their demand so as to maximize their utility. This is also quantity adjustment as it is explained by Jevons himself in Figure 7 (IV.21-22). In normal state of affairs, it is not the final utility which determines value but value determines the level of final utility of all individuals.

In this point one may point out that Jevons's understanding of Ricardo-Mill system was very superficial. He could not understand the logical structure of Mill's reasoning. Mill conceded that the cost of production ceases to operate when the quantity of production is predetermined. That was the case of complete specialization. In the two country, two commodity case, complete specialization was inevitable when one wanted that both countries gain from trade. But, as we shall see in the next section, the situation changes once we consider the case where the number of commodities exceeds number of countries.

As a conclusion, one may say Jevons depended totally on Mill's solution without understanding why Mill was obliged to consider the special case where the production is predetermined. Except for those non reproducible goods like pictures of Picasso or Klimt, the capitalist economy is determined primarily by the conditions of productions and not by subjective utilities. In spite of all these misunderstanding, it is a fact that Mill's "solution" gave Jevons an opportunity to propagate his highly subjective explanations of economies.

8.2 Alfred Marshall

Contrary to Jevons, Marshal acknowledges general intellectual debt towards the classical economics. What we have to do here is to know if my specific conjecture is applicable to Marshal. It is to ask whether Mill's "solution" gave a crucial impetus to the generation of Marshall's economics. The accompanying question is, if my conjecture is fundamentally right, how this impetus concretely worked in the making of Marshall's economics. This is also a tremendous subject matter and I have to leave main parts of these questions to the historians of economics theory. I present here only a hint which may help to consider my conjecture and related questions.

A fact is clear. Marshall started his intellectual walk from reading J.S. Mill. As Schumpeter (1954, p.804) put it, Marshall's "acquaintance with economics commenced

with reading Mill in 1867–8 (Memorials, p.10)." He worked from early days of his research on trade problems. In contrast to Jevons, Marshall emphasized the continuity of economics. In fact, in the Preface to the 1st edition of *Principles* he wrote:

> As, in spite of the great differences in form between birds and quadrupeds, there is one Fundamental Idea running through all their frames, so the general theory of the equilibrium of demand and supply is a Fundamental Idea running through the frames of all the various parts of the central problem of Distribution and Exchange. (Marshall, 1890, Library P.7)

This part is cited 32 years later by himself in the Preface to Marshall (1923). It proves that Marshall conserved this thought throughout his life.

The quotation is remarkable in two points. If the metaphoric comparison between birds and quadruped indicates classical and neoclassical economics, the comparison means that the basic theoretical structure of economics remains the same. Marshall thinks this continuity is preserved by the "the general theory of the equilibrium of demand and supply." The first point of interest is, then, how Marshall recognized the fundamental difference between classical and neoclassical economics. The second point of interest is how and perhaps why Marshall thought that the theory of the equilibrium of demand and supply was the fundamental idea which runs through history of economics from classical to neoclassical economics.

As for the first point, I have yet no textual evidences. We must inquire how Marshall estimated Ricardo's objection to the demand and supply theory. The second point is closer to our main concern. When and how did Marshall come to think like this? One possibility immediately remind of is the Mill's conclusion that the principle of demand and supply is "anterior" to the principle of cost of production. If there are no other explicit evidences which contradict to this interpretation, this is most hopeful hypothesis.

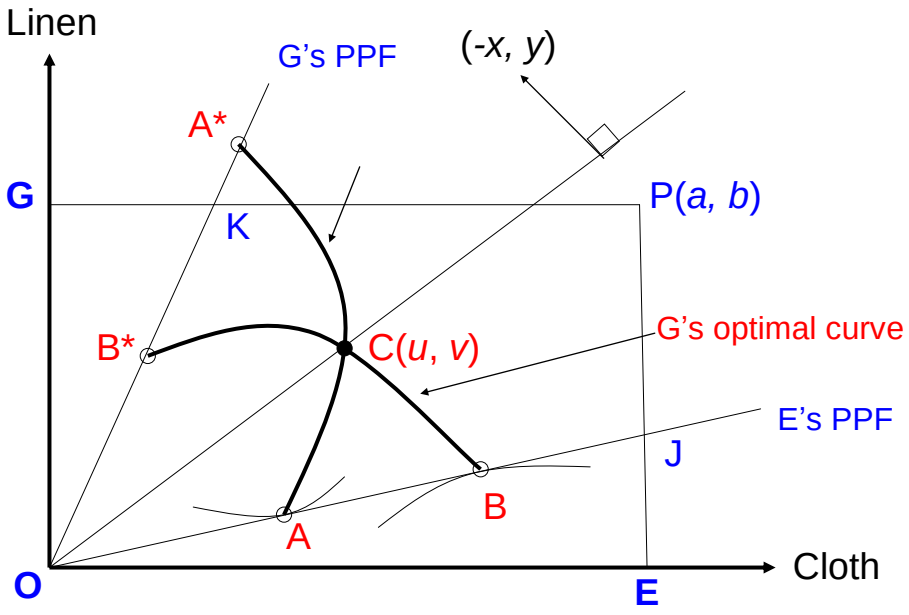
In order to strengthen this hypothesis, let us examine the logical structure of Marshall's theory of international trade, which may reveal a Marshall's deep debt to the Mill's "solution." In his early days as researchers Marshall wrote several papers on foreign trade and value questions and compiled them to be published as his first book Marshall (1879). I had no occasion to examine Marshall (1879). Let us confine our examination to only two papers in Marshall (1923): Chapter VI of Book III and Appendix J. Chapter VI is a general introduction to the theory of international trade and Appendix J is a

mathematical exposition of the content of Chapter VI. In the note 1 to this Appendix J Marshall testifies that this paper was chiefly written between 1869 and 1873. It means that Appendix J forms one of the earliest layers of Marshall's research works.

In this paper (Appendix J), Marshall treated the same problem as Mill's and started to analyze demand-offer curves. This is thought to be the first step to Marshall's famous scissors diagram or Marshallian cross analysis[12]. It was a big jump, but one on Mill's step. As Humphrey (1995, p.52) appraises, "[i]t was Alfred Marshall who took the crucial step of translating Mill's instructions into geometry and thus invented the diagram that bears his name." Lineage from Mill to Marshall is clear.

To examine the deeper structural similarity between J.S. Mill and Marshall, it will be convenient to try to rebuild Mill's "solution" and his explanation in a diagram. One of such trial is shown as Figure 3. I do not claim that this is a loyal reconstruction of J.S. Mill's thinking process. This is rather a construction how Marshall would have understand Mill's explanation[13] It is a bit complicated but I think it expresses all the necessary elements of Mill's analysis in one shot.

Figure 3 Mill's "Solution": A Graphical Rebuilding



[12] I am not claiming that cross diagram was fist introduced by Marshall. For early uses of cross diagrams, see Humphrey (1992).

[13] A detailed analysis of Mill's thinking process is made in Yoshii (2014).

Two countries E and G are producing cloth and linen. The diagram is in a sense a juxtaposition of two coordinate systems. One represents E's production and consumption points with its origin at E. Another represents G's production and consumption points with its origin at G. E's production possibility frontier (PPF) is given by a thin line starting from the graph origin O (E's production possibility set is the triangle OEJ). G's PPF is given in the same way (The production possibility set is the triangle OKG).

Country E has comparative advantage in cloth and country G has comparative advantage in linen. The length OE is given by the quantity of cloth a when country E specializes in the production of cloth. The length OG expresses the quantity of linen b when country G specializes in the production of linen. When both countries specialize in their more competitive products, the world production is (a, b). A point inside the rectangular box OEOG is represents an allocation of produced products. For example, if the coordinate of a point Q is (u, v), then E's share is $(a-u, y)$ and G's share is (u, b－v).

A point in the box can represents demand and offer of a country. For example, a point $Q(u, v)$ stands for the transaction that country E supplies quantity u of cloth to country G and receives quantity v of linen from G. The same point can represent a transaction that country G supplies quantity v of linen to E and receives quantity u of cloth from E. Thus each point inside the box represents an offer of country E and an offer of country G. If the offer points of both countries coincide, demands and supplies of two products are consistent and one can realize offers of both sides. If the offer points of two countries differ, two offers are not consistent and no transaction can satisfy both offers.

It is useful to know, for later analysis, what will happen when two offers are not consistent. Take an example that country E offers point $A(u, v)$ and country G offers point $B(u', v')$. If we consider a net offer, point A means that E offers $(u, -v)$ and point B means that G offers (－u', v'). If we see fix our point of view to E side, this stands for a net offer $(u-u', v'-v)$. In the graph, A lies left to and lower than B, offer combination A and B means that the net offer of cloth is negative and the net offer of linen is positive. In this case, exchange ratio will be changed in favor of cloth. Offer combination like A* and B* means positive cloth offer and negative linen offer and it is probable that exchange ratio will be changed in favor of linen.

Now basic explanations of the diagram are finished. Of course, John S. Mill did not used the terms like production possibility set or frontier. But he was aware that the frontiers of two countries have different slopes, for he knew two countries have different production prices or domestic values. Slopes themselves do not express prices. Values are expressed as direction which is perpendicular to the production frontier. The fact that country E has a comparative advantage in cloth and country G has a comparative advantage in linen is expressed by the fact that the box has a space which is not covered by one of two production possibility sets. Mill thought that each country (or the people of each country) will choose an offer point when (relative) values are given.

Suppose now two countries completely specialize in their competitive products. Cloth produced by E is the quantity a or length EO. Linen produced by G is the quantity b, or length GO. The point O represents production for both countries when they are completely specialized. Suppose a set of values is given. For example, suppose that it is proportional to the production prices of country E. Then half line OJ (or PPF for E) represents budget constraint for both E and G. The difference lies in this. Any point of the lower region of the budget constraint satisfies the budget balance for E, whereas any point of the lower region satisfies the budget balance for G.

Mill thought that each country chooses an offer point on the budget constraint line. He did not specify how this point is related to utility. Jevons mainly analyzed this point. I have drawn a thin curved line which is tangent to line OJ. If the curve is an indifference curve for E, then point A is the offer point of E, for it is the point where E's utility is maximized. Take G's indifference curve which is tangent to line OJ. Then the point B is the offer point for G. Take any values between domestic values of two countries. By the same reason, one can find two offer points for E and G on a half line which emanate from O. The diagram 'or Figure 2 in this paper) Jevons used to illustrate how people maximize their utility is concerned to this part of analysis. Here we have drawn a indifference curve, but we can draw a similar figure if we want.

Now let us consider that the values changes. If half line turns anti-clockwise, the relative value of lines goes up. Imagine one raises gradually the relative value of linen from half line OJ to half line OK, at each half line in between has two offer points, each for one country. If we assume that these offer points move continuously, we get two curves which start from A and B and arrives A*and B*. These are the offer curves Mill

would have imagined when he remarks "[t]he demand for a commodity, that is, the quantity of it which can find a purchaser, varies, as we have before remarked, according to the price" (Mill, 1848, III.18.10). Mill did not explicitly assume that these curves are continuous. However, we must say this is tolerable, for at the time when Mill (1848) was published it was normally believed that all functions are continuous[14].

I do not think that Mill was worried at the existence of an equilibrium point. But if we examine this question, Mill was concerned with conditions which were more valuable than Jevons and even Marshall were. If two offer curves are continuous, one can easily deduce existence of an equilibrium point. One sufficient condition for it is that the order of offer points changes from A to B on half line OJ to B* to A* on half line OK, always starting form origin O. This is but a variant of the intermediate value theorem. Without using the intermediate value theorem, it is intuitively evident that two offer curves must cross at least at a point. Let the crossing point be C. The point C(u, v) gives the equilibrium. The equilibrium prices are given by the direction normal to half line OC i.e. v/u and transaction between E and G are given by the vector (u, v) i.e. E supplies quantity u of cloth and G supplies quantity v of linen.

Presumably John Mill did not know intermediate value theorem. Although this theorem is intuitively correct, its formulation is difficult when the notion of continuity was still unclear. All he could do was to infer relying on intuition. In spite of all these disqualifications, Mill's construction is in a sense better than Jevons's method of counting equations and unknowns. In the latter method, no further analysis is possible than believing the existence of a solution. On the other hand, Mill's construction leads to other kind of observations than the existence of a solution. For example, what happens when B comes between O and A? In this case, country E produces both cloth and linen and international values are proportional to the domestic values of E. Similar observation is possible when A* comes between O and B*. In fact, these cases are explicitly examined in note 54 in Chapter on International Trade (Mill, 1847, III.18.54 note).

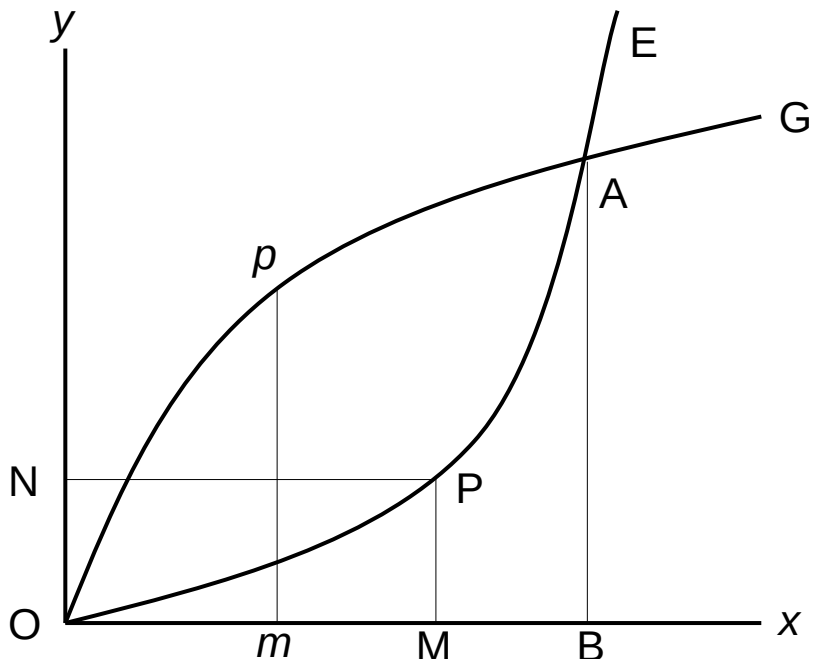
Comparison of Mill's and Jevons's methods in proving the existence of a solution, however, is not our aim in this subsection. Our mains interest here lies in how Marshall inherited from Mill. If we compare Figure 3 and figures in the Appendix J, we can

[14] Continuity of a function was first defined in a modern sense by Dirichlet in 1837, but conceptual confusions around function and continuity continued up to 1870's. It is even doubtful if Mill had a concept of a function in modern meanings. See O'Connor and Roberson (2005).

confirm close resemblance. Take Fig.1 in particular. It is here reproduced as Figure 4.

If we compare Figure 3 and Figure 4, it is quite easy to know the specific difference and the family resemblance. Ignore the box OEQG, Mill did not explicitly talked about the production possibility set. Marshall inherited this point from Mill. Difference lies in the shape of two offer curves. Marshall starts his curves from the origin. Mill was not explicit in this point, but as he was aware that each country has his inherent prices and consumptions. Hence, if he described curves, he should have started his curves not from the origin but somewhere from points A and points B (as in Figure 3). So the dotted parts of the Marshall's curves are absent in Mill's virtual curves. This difference corresponds to the fact that Marshall did not specify the limits that the exchange ratio must have from below as well as from above. Why did Marshall forget that the international exchange ratio comes between tow country's domestic prices? It is probable that when Marshall started his analysis in international trade, he had forgotten that trade is in fact supported by each country's productions. If this supposition is true, this means that Marshall went one step further into the analysis of pure exchange economy.

Figure 4 (Fig.1 Marshall's graphical presentation)



Marshall's mathematical examination is much better than Jevons. At least Marshall examines the shape of two offer curves and he proves the uniqueness of equilibrium when two offer curves are normal (Proposition 8, p.458). He also examines the stability

of the equilibrium and discusses the case (when curves are exceptional) where more than two equilibrium points exits. On the contrary, he discusses nowhere the existence of equilibrium point itself.

Marshall was correct in one sense. In his construction, there is no warrant that equilibrium pint exists. We should distinguish two different situations. One is the case where tow curves never intersect. An example is given in Figure 5b. If two curves have asymptotes which are parallel to each other as indicated in Figure 5b, two offer curves never come to intersect however they are extended. Another case is indicated in Figure 5c. Let two curves intersect at a point $C(u, v)$. If the intersect comes out of the production range, or if $u > a$ or $v > b$, the trade is not feasible for the production of either one of products is too big to be producible by a country.

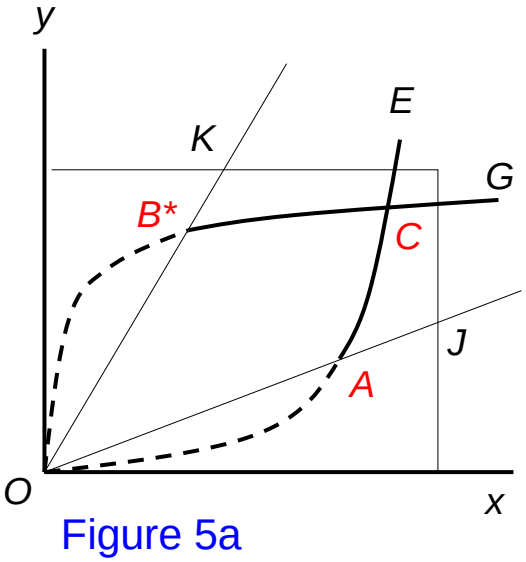


Figure 5a

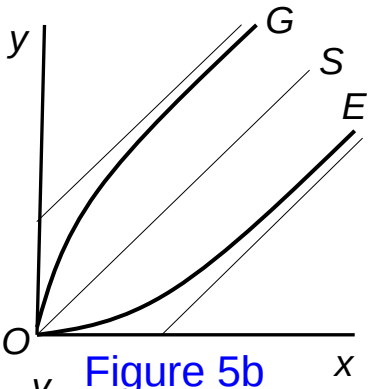


Figure 5b

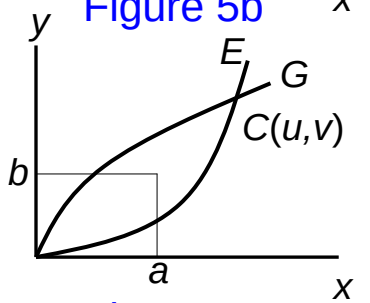


Figure 5c

Appendix J, which is assumed to be the starting point of Marshallian cross, has omitted elements related to production. In this sense, the shift from economics of production to economics of exchange progressed further than J.S. Mill. We cannot say anything firm about the influences from Jevons. In the first paragraph of Section 8, Marshall writes like this:

> And, though its direct applications are only to some theoretically conceivable issues of the trade between two isolated countries, it can be translated into the terms of any sort of bargains between two bodies, neither of whom is subject to any external

competition in regard to those particular bargains.

This phrase and possibility of translation remind us Jevons's "trading body." If we don not know when this phrase was written, two possibilities are possible. It is possible that Marshall inserted this phrase after he read Jevons (1871). It is possible that Marshall has written this independently. At either case, we can confirm that Marshall interpreted J.S. Mill's "solution" as a general situation to start when one wants to construct whole system of economics. This is nearly a completion of paradigm shift from economics of production to economics of exchange.

8.3 Francis Ysidro Edgeworth

In the case of Edgeworth, it is difficult to distinguish direct influences from J.S. Mill and indirect influences from Mill through Jevons and Marshall[15]. In the case of Edgweorth, too, there exist very few direct evidences. But it seems there is a strong circumstantial evidence.

Edgeworth was continuously interested in international trade questions and left several papers including Edgeworth (1894). The third part of the latter is a detailed survey of "classical" pure theory of international trade. The modifier "classical" here means "after Ricardo and before Marshall" (a bad custom which was conserve up to Chipman (1965) days, for the majority of works examined are neoclassical in their orientation.). His first and most famous *Mathematical Psychics* (Edgeworth, 1881) contains diagrams which marked a step to now called Edgeworth Box Diagram. In that book, there are two explicit mentions are made to Mill's *Principles* (Mill, 1848) but no mentions on the International Value chapter (Book III, Chap. 16), which Edgeworth later called "great" and "stupendous chapter" (Edgeworth, 1894, I, p.38; III, p.607.). It is not certain but possible that at the time Edgeworth was writing *Mathematical Psychics*, he had not read the chapter or Mill (1844) and did not know Mill's conclusion that, in view of inapplicability of cost of production principle, "we must revert to the principle of demand and supply" (Mill, 1844, I.19; Cited above at the end of previous section). It is also possible that Edgeworth intently avoided mentioning International Value chapter, as he was planning to write Edgeworth(1994) or other papers whose subject is more cloth to Mill's work on international trade. On the other hand, Edgeworth (1894) does not reveal any special indebtedness to Mill's chapter. Edgeworth treats Mill as one of great antecedents who contributed to the development of international trade theory.

[15] Edgeworth acknowledges that he owes to Marshall's unpublished paper at that time (Edgeworth 1881, II. p.443).

Apparently, there seems to be no deep influence of Mill's "solution" on the formation of Edgeworth's economics.

Despite of these negative facts, I am still attached to my conjecture that Edgeworth was also influenced in a deep layer of theory by Mill's solution.

Mathematical Psychics contained superb ideas which cannot be confined to be a mere analysis of demand and supply. It was the first analysis of two person exchange economy, which was later developed as the theory of the *core of an economy* (Debreu and Scarf, 1963). In one aspect, it contained much superior theory of exchange than the demand and supply equilibrium, for Edgeworth's research program was to start from a two person trade game and proceed to a many person trade game. Introducing competition in this way is much more concrete than the standard perfect competition concept, i.e. the state that all agents behave as price takers.

How did this idea come to Edgeworth? Edgeworth's idea of two-person, two-commodity exchange economy is very close to the economy Mill examined in his international value question. I do not deny that a minimal model of exchange would be two-persons, two-commodity economy. But, was it a common sense to talk about two persons when Edgeworth started to study economics?

Any barter or trade is a transaction between two parties. An exchange is a transaction of two different things between two sides. Even though, it is not common to set a situation of two persons and two goods and to search an exchange rate. This is what Edgeworth started to do. It would be a bold and difficult imagination to believe that one can analyze two-person two-goods bargaining and arrives to fix a rate of exchange. In addition, Edgeworth simply imagined two persons who were in possession of two kind of goods without any indication how they came to own them. In the long tradition of classical economics, it was customary to imagine two persons who work to obtain their games or products. This is in essence an economy of production. How and why did Edgeworth come to analyze pure exchange economy? Even if it is possible, how did he believe that this start can lead to an understanding of a complex economy as that of 19th century England? If there was no Mill's solution, it would be difficult for Edgeworth to come to consider contract curve. In this sense, I believe that Edgeworth was influenced by Mill in a deep layer of theory making.

Another circumstantial evidence is the so-called Edgeworth Box Diagram. Now this diagram is adopted usefully to illustrate the two country two commodity trade situation and how the exchange rate is determined. The original diagram, described by Edgeworth lacks at least two important elements: box itself which expresses the predetermined quantities of product, and a line which expresses exchange ratio. These elements became added well after Edgeworth. Even though, Edgeworth Diagram, together with his contract curve, shows well the future development of his diagram. Difference lies but it is the question of elaboration. One can say that Edgeworth diagram was the conceptual construction well adapted to explain trade between tow countries. It is difficult to say that this is a pure coincidence.

§ 9. How is Mill's "solution" misplaced?

Now it is time to argue Mill's "solution." Was it a really solution to the Ricardo's problem of constructing a theory of international values. Was Mill's "solution" a unique possible solution to the theory of international values? By no means! Mill's "solution" was an exceptional case. Mill and many other economists in international trade theory mistook this exceptional case as a representative situation of international trade. Let me explain first by a figure.

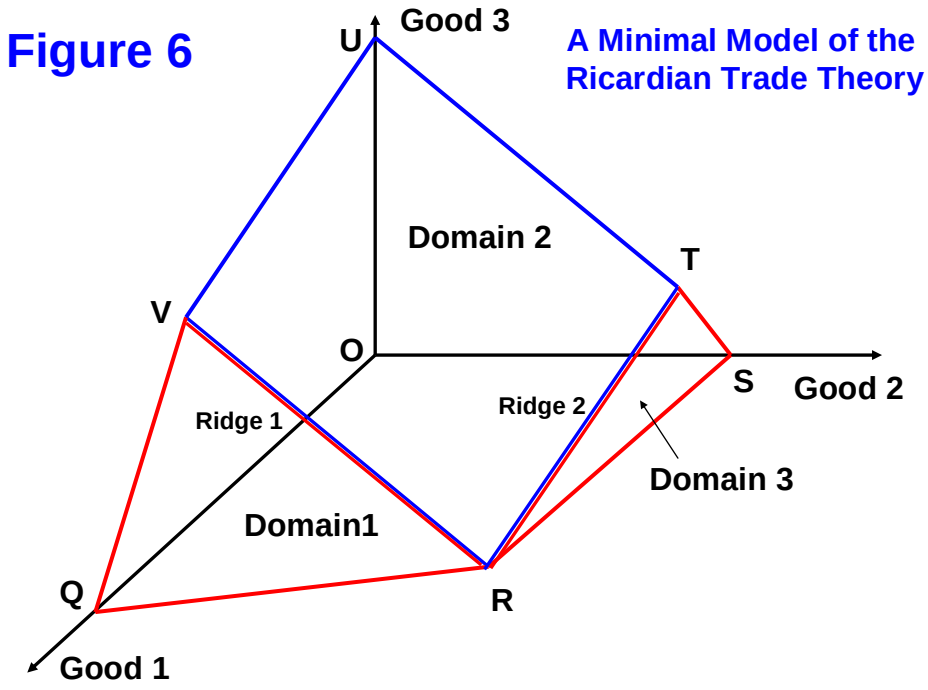


Figure 6 presents the world PPF in the case of two-country, three-commodity model. It

is named minimal model of the Ricardian trade theory. I will justify this denomination soon.

The world PPF of Figure 6 consists of three (open) domains, seven ridges and six vertexes. The first observation is that there is no vertex in the interior of the positive orthant. No point like K in Figure 2 exists. We named such a point Mill-Jones point[16]. Formal definition of Mill-Jones point is any edge point of the world PPS in the interior of the positive orthant. Coordinates of such a point must be all positive and extreme point of the world PPS (We omit the modifier "world" here after).

As we have observed there exists no Mill-Jones point, or interior edge point in Figure 6. One may think that this is a mere incidence, but it is not. Any trade model has no Mill-Jones point if the number of commodities exceeds the number of countries. An interior edge point represents a situation of complete specialization. This means that one country has only one commodity which is competitive in the world. Then, there is a one-to-one correspondence from set of countries to the set of commodities. Therefore, if a Mill-Jones point exists, the number of commodities is equal to or less than the number of countries. In Figure 2 case, numbers of countries and commodities are equal and a Mill-Jones point exists as a general case.

There are edges in the boundary of PPS. At these points, one or more coordinate vanishes. It means that one or more commodity is not produced at all. These are degenerated cases and we do not consider them, for they are the situation where the number of produced commodities is not greater than the number of countries.

Mill-Jones points or interior edges do not exist when the number of commodities is greater than the number of countries. This theorem went unnoticed nearly 150 years after J.S. Mill. But it has an important consequence. We can count about 200 countries or economies in the world. The number of commodities, although it is difficult to count them exactly, easily exceeds millions. For example, a price book of the Gosplan in the former Soviet Union, it is said, contained more than 20 millions of items. We may assume any developed economy trades and produces more products than that. We may safely assume that the number of commodities is bigger than the number of countries.

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[16] Some call this point "Ricardo point" or "Ricardo's Limbo point". Because these are not a suitable naming, I do not adopt this naming. The reason of my naming is given later.

Non-existence of Mill-Jones points means that there is no pint at which prices can move freely within an open range. People after Mill continued to try to determine prices in the assumption that they have some margin of free movement, but there is no such possibility when the number of commodities is bigger than the number of countries.

What happens in a point other than Mill-Jones point? If we see Figure 6 again, we easily know that most point of the PPF lies in either of three domains. In Domains 1 and 3, the prices are proportional to those of a country. John Mill excluded this situation considering that if trade continues, both countries must have certain gains from trade. In the two-country, two-commodity case, such a point is unique. It is point K of Figure 1. In case of two-country, three-commodity case, we have a wide Domain 2. Any point in Domain 2 has a system of international values which remains constant as far as the point stays in Domain 2. This system of international values is different from either of two countries' domestic systems of values and two countries (or people in both countries) can enjoy gains from trade.

There is a possibility that the world production comes on one of two ridges RV and RT. At a point on ridge RV, for example, the prices can vary but they must remain perpendicular to the ridge RV. The degree of freedom of those prices is only one dimensional. Moreover price changes have no effects to move productions and therefore supplies along the ridge. In this sense, price adjustment does not work at anywhere.

Domains 1 and 3 are the cases which are sometimes studied as big country cases. Although Domain 2 in Figure 6 stands in a similar situation like point K of Figure 1, they present very different characteristics. At point K the prices moves freely (the degree of freedom is the same as number of commodities) and world production is fixed. At any point in Domain 2, the price system remains constant and world production can change freely in the domain. These characteristics are quite similar to classical value theory. There is only one value system and supplies can be adjusted to any effective demand as far as it stays in Domain 2.

Now the whole picture becomes clear. Mill examined a two-country, two-commodity case believing that the model gives a representative situation and fell in an unexpected trap. He did not imagine that the situation changes very much when the number of commodities is bigger than the number of countries. If we admit that in a normal case the number of commodities exceeds the number of countries, the minimal model of

international trade should be two-country, three-commodity case. This is the reason that I call Figure 6 a minimal model of the Ricardian trade theory.

Once an intellectual tradition establishes, it continues for long time in some cases. Phlogiston theory in chemistry survived more than a century until Lavoisier establishes oxidation theory of combustion. Aether hypothesis as conveying medium of light continued more than three centuries until Einstein announces his special relativity theory. Sometimes an erroneous hypothesis came to be even strengthened by a most proficient scholar. In the case of Aether, Newton played such a role by assuming "aethereal medium" to explain refraction and diffraction. Mill's tradition to examine an interior edge point or Mill-Jones point continued until very recent days. I myself have been a victim of this tradition when I was trying to write a paper which took form in Shiozawa (2007). This paper was intended to be a successor to my paper in 1985 (written in Japanese) and more than 20 years I tried continuously to find theorems which might give sufficient conditions for the existence of an interior edge points in the models of input trade.

The tradition was strengthened by mathematical study of 1950's. McKenzie wrote three-odd papers on Ricardian trade model and Jones followed him. Their works marked a new era as their analysis was based on the new mathematical tool developed by the activity analysis which was a topical trend at that time. Jones (1961) discovered a formula I mentioned in Section 7. It was a beautiful generalization of Mill's formula for comparative advantage. In Mill's case, country 1 specializes in commodity 1 and country 2 specializes in commodity 2 if two ratios satisfy the following estimation:

$$a11 / a12 < a21 / a22. \qquad (9\text{-}1)$$

Here a ij are labor input coefficients. The above condition is equivalent to

$$a11 / a21 < a12 / a22. \qquad (9\text{-}2)$$

These two conditions are in this turn equivalent to

$$a11 \cdot a22 < a12 \cdot a21. \qquad (9\text{-}3)$$

Then the above three equivalent conditions (9-1), (9-2) and (9-3) are also equivalent to the existence of positive numbers $w1$ and $w2$ which satisfies the conditions

$$w1 \cdot a11 < w2 \cdot a21 \quad \text{and} \quad w2 \cdot a22 < w1 \cdot a12. \qquad (9\text{-}4)$$

This equivalence theorem can be demonstrated as follows. If positive numbers $w1$ and $w2$ which satisfy (9-4) exist, multiplying both sides of the two inequalities, and eliminating $w1 \cdot w2$ from both sides, we get (9-3). The converse holds too. In fact, if

assume condition (9-2) holds, take positive numbers $w1$ and $w2$ in such a way that

$$a11 / a21 < w2 / w1 < a12 / a22.$$

Then we can derive the first inequality from the left inequality of (9-4) and the second inequality from the right inequality of (9-4).

Suppose that two positive numbers $w1$ and $w2$ are wage rate for countries 1 and 2 respectively. Then the left inequality of (9-4) means that the cost of production of commodity 1 in country 1 is cheaper than the cost of production of commodity 1 in country 2. In other words, the production of commodity 1 in country 1 is more competitive than in country 2. In the same way, the right inequality of (9-4) implies that the production of commodity 2 in country two is more competitive than in country 2. With wage rate $w1$ and $w2$ which satisfy (9-4), we get an economy in which country 1 specializes in commodity 1 and country 2 specializes in commodity 2. Thus the above equivalence theorem implies that real cost and money cost approaches are in fact equivalent in the case of two-country, two-commodity case.

This interesting theorem was generalized by Jones (1961) for the case of N-country, N-commodity case. We present the theorem in a little different form than that given by Jones:

[Jones Theorem]

Let aij be labor input coefficients for country i to produce commodity j. Suppose there are N countries and N commodities. Then the following two conditions are equivalent:

(1) There exist positive wage rates $w1, w2, \ldots, wN$ which satisfy

$$w1 \cdot a11 < w2 \cdot a12,\ w3 \cdot a13, \cdots, wN \cdot a1N;$$
$$w2 \cdot a22 < w1 \cdot a21,\ w3 \cdot a23, \cdots, wN \cdot a2N;$$
$$\cdots$$
$$wN \cdot aNN < w1 \cdot aN1,\ w3 \cdot aN3, \cdots, w(N-1) \cdot aN(N-1).$$

(2) For any permutation τ of N indexes which is different from identity,

$$a11 \cdot a22 \cdot \cdots \cdot aNN < a1\,\tau(1) \cdot a2\,\tau(2) \cdot \cdots \cdot aN\,\tau(N).$$

The second condition can be translated that left side product is the strict minimum among permutation products of the form of the right side. The condition (1) is equivalent to the existence of internal edge or positive extreme point.

We do not prove the theorem here. Derivation of condition (2) from condition (1) is easy.

If we chose a suitable pairs of inequalities and multiply them, we get one of inequalities of condition (2). The converse is rather difficult. With a help of Helly theorem, we can prove it by mathematical induction with regards to integer N (Shiozawa, 2011).

One of Jones (1961)'s main results was Jones Theorem (or its equivalent theorem formulated in a different version). If we replace ordering of names of commodities, condition (2) only means that there exists a strict minimum among permutation products. This is valid for almost all choice of coefficients unless they have two minimal permutation products. In that case, a point of a point of complete specialization exits in the PPF. In other words, there exists Mill-Jones point in general.

Jones theorem gives a sufficient condition for the existence of a Mill-Jones point. The theorem also implies that if Mill-Jones points exist at all, they are unique. We can thus observe a keen interest on Mill-Jones point from J.S. Mill to Jones and others. However, as we have observed, when the number of commodities N is bigger than the number of countries M, no such point exists. If M=N, there exists at most one Mill-Jones point. If M>N, there may exist many Mill-Jones points. However, we should well acknowledge that these points only exist in an extremely unrealistic situation that $M \geqq N$.

Now it is clear that we should examine not an interior edge but points in a most general situation. If we return to our minimal model and Figure 6, such points are interior points in domains 1, 2, 3. In the terminology of convex polytope theory, these domains are called facets. Facet of a polytope is a common set of the polytope and its supporting half space (i.e. a half space which contains no points of the polytope except boundary points). In the case of N-commodities, the PPS is in general N-dimensional set and a facet is a boundary polytope with dimension $N-1$. Any point of the boundary is at least contained in a facet. A regular domain of the PPF is an interior of a facet. Any point in a regular domain is included in a unique facet.

At a point of a regular domain, a set of prices is uniquely determined up to a scalar multiplication. It is perpendicular to the facet and remains constant as long as the point stays in the same regular domain. An important lemma shows that this set of prices is associated to a unique set if wage rate for all countries. Thus in a regular domain, wage-price system is uniquely determined (up to scalar multiplication). We omit this modifier hereafter. When a set of wage-price system is determined, each country has at least one commodity which is competitive in the world market. Any point of a positive

boundary of the PPS can be expressed as a sum of competitive productions.

These observations about points in a regular domain hold for a very wide circumstances. This permits us a construction of new theory of international values as a generalization of classical theory of values. An exact formulation requires mathematical preparations and proves. All those complications are left for my book (Shiozawa, 2014, sorry in Japanese). In the next section, we give only a brief description of the new theory.

We may ask if there were no trials in the direction of a new theory. Yes, of course. McKenzie's Princeton teacher Graham was the most famous dissident to the mainstream tradition. Even before Marshall and Edgeworth, there were three notable exceptions: Nassau Senior (1790-1864), Hans von Mangoldt (1824-1868) and Henry Sidgwick (1838-1900). All three cases are interesting but require a long story for each. I have to omit entering to this topic. Interested readers are requested to check by themselves their books listed in the bibliography.

These are names I have some information of. It is possible that there are many others if we start to search with a new eye. An interesting anecdote is that Marshall once examined two-country and many-commodity case and nearly arrived to recognize the existence of wide ranges of constant prices (Marshal, 1922, Appendix H).

§ 10. A Brief Description of the New Theory of International Values

In this section, the new theory of international values is briefly described. The core idea of the new theory is to pay attention to the facets of the PPS (production possibility set). The (relative) interior of a facet is called regular domain. The PPF (production possibility frontier) of a trading economy is composed of finite number of facets. Any point of the PPF is either in a regular domain or on the boundary of a regular domain. This means that almost all points of the PPF are contained in a regular domain. In the two-country, three commodity case, the PPS is given by Figure 6. The PPF is composed of three facets and all points except on a ridge are contained in one of three regular domains. Figure 6 is a guiding example what happens in a trading economy.

This core idea holds for very large class of trading economies. The new theory treats the case of M-country, N-commodity case for any positive entire numbers M and N. It is not restricted to a labor input economy. In fact, the new theory treats the situation where

commodities are produced by means of commodities (Sraffa economy). Traded commodities include intermediate products or input goods. We assume a situation where exist a finite number of production techniques for the production of the same commodity. In other words, choice of techniques takes place. As a first step, we assume that transport cost is 0. Finally, the theory can be generalized to the economy where transport requires positive costs.

Formerly we assume the following situation:

(a) There are M countries.

(b) There are N commodities. They are traded freely between countries and transported without cost.

(c) Each country has a homogeneous labor power whose quantity is fixed as a first step.

(e) For each country a set of fixed coefficient production techniques are given.

(f) Production techniques are assumed to be simple, i.e. only one product is produced by one production technique. For simplicity we assume that each country has at least one productive system of techniques.

(g) To produce any good in any country requires a certain (positive) amount of labor of that country.

An economy which satisfies conditions (a) to (f) is called *Ricardo-Sraffa trading economy*.

A set of different commodities is denoted by an N-row vector $\mathbf{x}$ and called commodity vector. As commodities can be transported freely without cost, we can treat them abstract of locations. Prices of any good are all equal anywhere in the world. A price vector will be denoted by a N-column vector $\mathbf{p} = (p(i))$, where $p(i)$ is the price a good i. The wage rate of a country i will be denoted by $w(i)$. A set of wage rates for all countries is denoted by a M-column vector $\mathbf{w} = (w(i))$. A value vector $\mathbf{v}$ is a set of wages and prices arranged in a row vector and composed of $M+N$ entries. Each entry indicates either a wage of a country or a price of a good.

We suppose there are in total H different techniques in the world. Techniques are numbered in a certain order but there is no need to enter in this detail. It is sufficient to suppose that this order is preserved for all expressions. The set of all techniques available in the economy is expressed by a matrix of coefficients. The set of all net production vectors belonging to a different technique is expressed by an $H \times N$ matrix A. labor input matrix I is a $H \times M$ matrix whose entries are 0 or 1. Each row vector of I

contains only one entry with value 1 which indicates in which country the production is made.

Each country has a certain quantity of labor power $q(i)$. The set of labor powers of the world is denoted by M-row vector $\mathbf{q}$. When each technique is operated at the level $y(k)$, the production activities of the world are denoted by $\mathbf{y} = \{y(k)\}$. The net material production of the world is $\mathbf{y}\,A$ and the total labor input of the world is $\mathbf{y}\,I$. The i-th entry of this vector expresses the labor input for the country i. Finally the production possibility set for a set of techniques Γ and for labor power $\mathbf{q}$ is given by $P(\Gamma, \mathbf{q}) = \{\mathbf{y} \mid \mathbf{y}\,I \leqq \mathbf{q}\}$.

A series of theorems can be derived:

Theorem 1 (Polytope of a Production Possibility Set)
When $\mathbf{q} > \mathbf{0}$, the production possibility set $P(\Gamma, \mathbf{q})$ has a non-empty intersection with non-negative orthant. This intersection forms a polytope in N-dimensioned space $\mathbf{R}^N$ and its boundary is composed of N-1 dimensioned closed facets.

Theorem 2 (Existence and Uniqueness of International Value, Full Employment Case)
For any Ricardo-Sraffa trading economy and a positive vector $\mathbf{d}$, there is a production activity vector $\mathbf{y}$ and a unique set of international value $\mathbf{v} = (\mathbf{w}, \mathbf{p})$ which satisfy the following conditions:

(i) $\mathbf{y}\,A = \alpha\ \mathbf{d}$ for some positive α.
(ii) $\mathbf{y}\,I = \mathbf{q}$.
(iii) $I\,\mathbf{w} \geqq A\,\mathbf{p}$.
(iv) $\langle \mathbf{q}, \mathbf{w} \rangle = \langle \mathbf{y}, \mathbf{p} \rangle$.

The value vector $\mathbf{v} = (\mathbf{w}, \mathbf{p})$ is uniquely determined when $\alpha\,\mathbf{d}$ lies in a regular domain and remains constant if $\mathbf{d}$ remains in the same regular domain.

For the proof of the theorem, see Shiozawa (2007) Theorems 5.2 and 5.6. More formal and rigorous proof is given in Shiozawa (2014, Chapter 5).

Each condition of theorem 2 has an explicit meaning. Condition (i) implies the existence of a world production which satisfies a predetermined world demand (equality of demand and supply). Condition (ii) implies the full employment for all countries (full employment condition). Condition (iv) implies the equality of the total wage of workers

and the value of net products (money flow condition). Conditions (iii) and (iv) imply that that productions are made by using competitive techniques only (profitability condition). Customary condition of zero balances of payments is not assumed here. But they are satisfied automatically when workers and firms keep budget constraint. If we limit ourselves to competitive techniques, condition (iii) holds with equality. This means that the value determined in this way satisfies a system of equations similar to those of Sraffa. We can say the new theory is an international version of the cost of production theory of value.

It is important to note that, in the above situation, relative wage levels of all countries are uniquely determined as a part of international values $\mathbf{v} = (\mathbf{w}, \mathbf{p})$. In this sense, the new theory of international values differs much from HOS theory. The latter assumes that wage rates of all countries are the same as the standard situation (factor price equalization theorem). HOS theory gives no hints for improving level of living standard. Contrary to this, the new theory permits us to analyze why wage rate differentials occur.

Theorem 2 proves an existence of full employment reproducing state. This does mean that the new theory cannot be used to analyze unemployment state. For example, we obtain next proposition:

Proposition 3 (Existence of unemployment)
In the same situation as theorem 2, assume that world demand $\mathbf{d}$ lies in a regular cone (i.e. if $\alpha\,\mathbf{d}$ is in a regular domain for some positive α) but does not lie on the PPF, then unemployment is inevitable when all productions are made by competitive techniques with regards to the international values satisfying conditions of theorem 2.

Usual discussions on gains from trade can be given (See Shiozawa, 2007, Section 5). What is more important than the usual gains from trade is the possibility of trade conflicts. A sudden opening of a country for international trade or a rapid shift for more liberalization may induce unemployment and other economic troubles and may cause trade conflicts. Adjustment processes are important and the new theory provides tools for analyzing these adjustment processes.

§ 11. Implications for Future Research

Above discussion can be summarize as follows. Ricardo left two rectification problems. One was to construct a theory of domestic values in a more rigorous and universal way. This problem was principally solved by Sraffa (1960). Another problem was to construct a theory of international values as a generalization or an extension of classical value theory for reasonably wide range of international trade situations. This problem was solved, I believe, by Shiozawa (2014).

Mill's "solution" was misdirected, for the situation Mill examined is extremely special and normally non-existent. Mill mistook it to be a general and typical trading situation. Nevertheless the impact of Mill's "solution" was big. Mill conceded that laws of demand and supply were anterior and more fundamental to the cost of production principle. Mill's "solution" provided a typical situation of pure exchange economy and that paved the way to the neoclassical economics with cooperation of all other factors.

The classical theory of value at the time of J.S. Mill had several weak points. First of all, the classical economics was a mixture of all sort of idea. Many of them were not defendable. It has a firm and solid core, i.e. the cost of production theory of value. After Ricardo two problems were left. Cost of production theory of value was not properly formulated yet. Lack of international value theory was another. We identified these as two rectification problems. Both of them were now solved. With these results, theoretical power balance between classical and neoclassical economics changes. It is now time to develop a new economics based on classical theory of values.

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