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Samuelson's Implicit Criticism against Sraffa and the Sraffians and Two Other Questions

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Samuelson's "A Ricardo-Sraffa Paradigm" (2001) is an implicit criticism against neo-Ricardians who could not provide any theory beyond the small open economy. The necessity of constructing a trade theory with traded intermediate products was evident and was challenged by both strands: neo-Ricardians and mainstream economists. The attempt failed and the theme has been abandoned since long. My recent paper shows that a Ricardo-Sraffa type trade theory, with traded intermediate goods, is possible. In view of this theory, it is shown that the Samuelson's Conjecture of Limited Substitution is false. The relevance of a new theory is illustrated by a recent debate on the gains from trade.

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1 Introduction

In 2001, Paul Samuelson contributed a paper titled "A Ricardo-Sraffa Paradigm — Comparing Gains from Trade in Inputs and Finished Goods" to the *Journal of Economic Literature*. He chose a rather old topic — Ricardian trade theory — but from a new viewpoint. He compared two scenarios in which we gain from trade. The first one is Ricardo's, and the second is supposed to be that of Piero Sraffa[1]. Ricardo viewed trade as an exchange of finished goods such as wine and linen. Sraffa initiated a new viewpoint that gains from trade are a result of the trade of input goods. In order to convince readers, Samuelson cites an appropriate example (See Figure 1) where he shows that the gain from input trade is three times higher than that from the trade of finished goods. Samuelson referred to this gain from input trade as the Sraffian bonus.

Apparently, this is in acknowledgement of Sraffa's 1960 book *Production of*

[1] I have used the word "supposed" because, for the best of my knowledge, Sraffa himself had never explicitly argued this question.

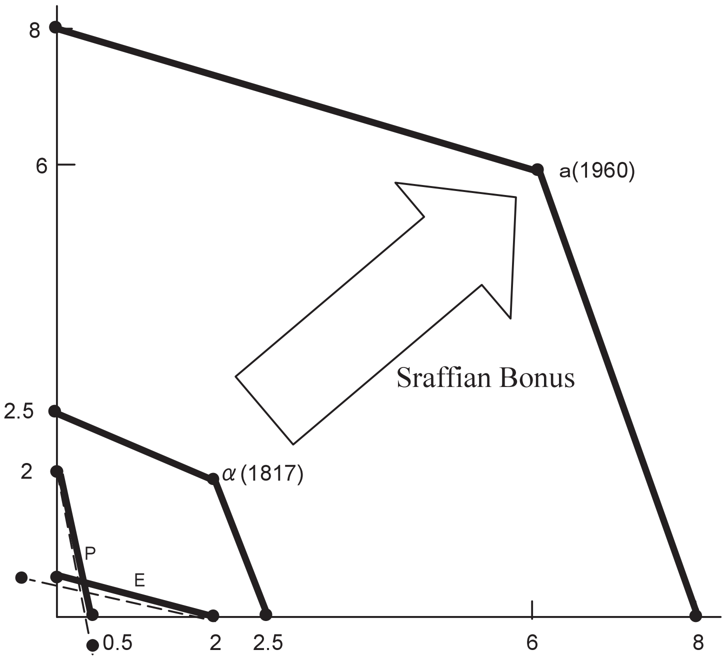

Figure 1: The Sraffian Bonus
Samuelson, "A Ricardo-Sraffa Paradigm," *JEL*, 2001

Commodities by Means of Commodities. Samuelson cited Böhm-Bawerk, Irving Fisher, Piero Sraffa, and Karl Marx as authors who can understand "how labor working with capital might enjoy higher productivities and higher competitive real wage rates." However, he attributed the concept of the input trade to Sraffa. This is somewhat unusual because Sraffa did not mention international trade in his 1960 slim book. The entire Sraffa story is Samuelson's invention. However, it is not surprising that Samuelson attributed the concept of gains from input trade to Sraffa.

In his book, Sraffa (1960) rightly shed some light on the importance of inputs. On the other hand, he was also the editor of the famous *Essays and Correspondence of David Ricardo* and presented a new interpretation of Ricardo's works. As a result of this and his own book, Piero Sraffa became the patron saint of a new research program, which led to the formation of a school termed by economists as neo-Ricardians or Sraffians. However, there was a peculiar lacuna in the 1960 book in that the author did not mention the Ricardian idea of comparative advantage. This lacuna was not filled by neo-Ricardians, although some of them worked mainly in the field of international trade (See Steedman 1979a and 1979b). They could have studied small open economies. Thus, the true enigma lies in (1) why neo-Ricardians could not recognize the importance of gains from input trade and (2) why they could not present a theory of international trade. In this new theory, the entire economy should be studied where commodities are produced by means of commodities, which are produced either in the country of production or in a foreign country.

In light of this unusual lacuna, Samuelson's acknowledgement must be considered as an implicit criticism of neo-Ricardians. Neo-Ricardians discussed the Heckscher-Ohlin-Samuelson theory (hereafter, the HOS theory) and strongly criticized it based on the fact that it relies on the concept of homogeneous capital. This was, in fact, a part of the famous capital controversy. In this controversy, neo-Ricardians together with English Cantabridgians were victorious, and Samuelson and his company was defeated (Samuelson 1966). However, neo-Ricardians could not formulate any constructive theory to replace the HOS theory. Although Samuelson did not mention the subject of neo-Ricardians, this silence itself was a form of astute satire.

Logically, this paper consists of four distinct parts. In Section 2, I show how the lacuna was conceived by neo-Ricardians and how several attempts were made. Despite their efforts, however, they could not go beyond the small open country context. In Section 3, I trace how the same topic was conceived by mainstream economics. In Section 4, based on my previous paper (Shiozawa 2007), I show that the neo-Ricardian lacuna was now filled by a new trade theory and that Samuelson's implicit criticism had been invalid since 2007.

In Section 5, I examine what Samuelson named the Conjecture of Limited Substitution. Samuelson's conjecture in a two-country two-good case is false, and I present a counterexample. Next, I show how to apply my general theory to these simple cases. I also discuss the conjecture in a three-country case, first provided by Ronald Jones. Although Samuelson's conjecture was false, a different form of "Limited Substitution Theorem" was obtained as part of the general theory of international trade with intermediate products, as I have indicated at the end of Section 4.

In Section 6, by way of a conclusion, I cite a recent controversy on the nature of gains from trade. In defending his position, Gregory Mankiw made it clear that he relies on the Ricardian trade theory, and argued that the Ricardian trade theory is more appropriate than the HOS theory in the world of the global capital movement, for the Ricardian trade theory assumes labor force as a unique immobile factor of production. However, there clearly exists a logical flaw in his contention unless a Ricardian trade theory that can treat input trade is provided. This small episode clearly shows that the topic discussed here is not only of historical interest but also has actual significance for present-day policy examinations.

2 A lacuna in Sraffa's book and its conception

Sraffa's 1960 book started that the circular nature of a production system necessitates abandoning the concept of capital as the primary resource. This very fact became the starting point of a famous controversy, which is sometimes called the Cambridge-Cambridge Controversy or Capital Measurement Controversy. Neo-Ricardians criticized the HOS theory. It was an application of the English Cantabrigians' criticism of neoclassical orthodoxy. The standard HOS theory assumes two factors of production: labor and capital. Ian Steedman was the main figure who led the battle in international trade theory. He presented a numerical example that showed the re-switching of techniques, and by consequence, capital reverse. He convincingly showed the logical flaw to the HOS theory. However, this criticism could not change the future flow of the international trade theory and the HOS theory remained the mainstream theory. The reason behind this state is obvious. Neo-Recardians could not present a new theory that could replace the HOS theory.

It is important to note here that Steedman and other neo-Ricardians who discussed international trade supposed a "small open economy" (Steedman, 1979, Introduction). In other words, they treated the economy of country A trading with the rest of the world. The term "small" implied that the influence of country A is negligible to the world economy. Other people (for example, Gontijo 2000) claimed that they succeeded in treating the entire world economy, in which two or more countries engaged in international trade. However, most of them adopted the "rest of the world" concept, in order to make their theory tractable. Few of them took the initiative to explore the case of many countries in which inputs are traded. A simple criterion to determine if they have succeeded in formulating an effective theory would be to examine whether they have succeeded in determining the possible patterns of trade and specializations. Despite this, in most cases, authors assigned a particular pattern of trade, but did not clarify why and how that pattern was possible[2)].

If Sraffa's main contention was the mutual dependence of costs and prices because of the circular nature of production, then an international trade theory of the Ricardian type should be constructed on the same principle. However, those Sraffians who dwell in a small open economy did not (and logically could not) succeed in constructing such a theory. It is clear that there is a lacuna in the Sraffian research programs. Steedman himself acknowledged this lacuna and wrote about the question twice (Steedman 1999 and Steedman 2002). Maneschi (2004) acknowledged the necessity of extending the "principles of the neo-Ricardian (or Sraffian) theory of international trade" and treated the case with input trade but his study is still limited to a small open economy analysis.

Samuelson's 2001 paper apparently acknowledged Sraffa's 1960 book, but at the same time, if read in depth, it was a satire on the inability of neo-Ricardians to recognize and fill the lacuna that might be the most important (and constructive) challenge for them. No neo-Ricardians seem to have reacted to Samuelson's implicit criticism. One of the few exceptions, are Baldone, Sdogati and Tajoli

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[2)] Sometimes, they claim that they are not interested in determining the pattern of trade and specializations. However, without solving this problem, a complete theory of international trade is not possible.

(2007). The authors claim that their example "builds on a Ricardo-Sraffa approach similar to the one developed by Samuelson (2001 and 2004)."

Pasinetti (2006) compared Samuelson and Sraffa and insisted that Samuelsonian and Sraffian economics run on two different tracks with different paradigms. Pasinetti admits that Sraffa's theory is "far from being a complete theory." However, he does not mention the possible construction of the Ricardo-Sraffa trade theory, although he cites Samuelson's 2001 paper.

Neo-Ricardians might have given up constructing a theory that answered Samuelson's criticism. However, it was a slightly hasty decision. As I show in Section 4, the task of constructing a Ricardo-Sraffa type general trade theory was already achieved by Shiozawa (2007).

3 A topic disappeared or abandoned?

Before presenting a short summary of my results, we shall see how the problem was conceived in the mainstream trade theory literature.

The necessity of formulating a new trade theory where inputs are traded was recognized in as early as the 1960s. Jones (1961) discovered a famous inequality formula by comparing the permutated products of labor-input coefficients. He tried to develop his results to "the case where intermediate products are freely traded." He only succeeded in treating cases where "the intermediate product structures are assumed (to be) the same in all countries." Another case that he was able to treat was that of an economy where "no products could be traded if they were to be used as intermediate products in some other country."

Jones's paper appeared to be independent of Sraffa's 1960 book; Jones was inspired by McKenzie (1954 and 1956). In these papers, McKenzie examined what he called the Graham Model, which is but a Ricardian model of a many-country, many-commodity case where labor is the unique input. Using the results of the activity analysis developed at that time, McKenzie succeeded in establishing some fundamental theorems such as the existence of equilibriums and the logical equivalence of production efficiency and profitability conditions. Furthermore, McKenzie added some observations to the case where intermediate products were traded. He observed that "it is not necessary to assume ... that the activities are *actually* integrated." He added that "it is sufficient that intermediate products do not appear in international trade" (McKenzie 1954, p. 166). If there exist several methods to produce a commodity, he hints that one can apply Samuelson's non-substitution theorem, given that joint production is absent. However, he was very well aware that the "simplicity [of the Graham model] is bought at the expense of prohibiting all trade in intermediate goods" (ibid. p. 180) and "as soon as trade in intermediate products is allowed, the problem loses its special simplicity" (ibidem. p. 179). McKenzie (1956) even stated in his introduction that "the basic flaw in the classical treatment of specialization is... the neglect of trade in intermediate products" (p. 56).

A few years after Jones's paper, Chipman (1965) wrote a comprehensive survey of the theory of international trade and published it in *Econometrica*. He paid special attention to the mathematical structure of the theory. After reviewing Jones's contribution, he wrote an independent section on Trade in Intermediate Products (§1.8). Chipman emphasized that McKenzie (1954) had stumbled upon the questions of intermediate products and discovered that "the introduction of trade in intermediate products necessitates a fundamental alteration in the classical analysis." Chipman added: "It is strange that this had not been noticed before, despite the fact ... that classical economists used intermediates products such as 'cloth' and 'linen' ".

Reiter (1953) and McKenzie (1954) remarked that "trade in intermediate products would enlarge the world production-possibility set" (Chipman, 1965, p. 509). Thus, the "Sraffian bonus" was discovered before Sraffa's *Production of Commodities by Means of Commodities*. However, as Chipman concludes in his section, "the most interesting consequences of the introduction of trade in intermediate products is ... the possibility of reversal in the pattern of specialization". This is that which McKenzie (1954, pp. 177–179) and Jones (1961, pp. 166–168) have emphasized. McKenzie's famous remark is that "there is nothing shocking to common sense in these results. A moment's reflection will convince that Lancashire would be unlikely to produce cotton cloth if the cotton has to be grown in England" (1954, p. 179).

Curiously enough, the theory of traded intermediate goods was not developed much after 1965. Sporadic works like Chipman (1971), Sanyal and Jones (1982), and Jones (1994, 1996) appeared; however, it appears that no Ricardo-Sraffa type general theory was presented by mainstream trade theory workers.

In 2000, Ronald Jones published a book under the title *Globalization and the Theory of Input Trade*, based on the Ohlin Memorial Lectures given in 1997 in Stockholm. Jones did not intend to overview the trade theory. He treated new problems that became important through increased mobility of inputs. In fact, the volume of input trade has increased and various forms of processing trade (including fragmentation and outsourcing) take place more widely than ever. However, Jones never mentioned any results with regard to the theory of international trade where the intermediate goods or inputs are traded. This fact shows the state-of-the-art theory of international trade. The general equilibrium framework is too general to be used as a basis for determining the pattern of specializations.

The interest in trade theory in intermediate products appears to have disappeared. At the turn of the century, international trade theory, whether it is neo-Ricardian or mainstream, lacked a theoretical basis when input trade is considered.

4 How can a new theory be constructed?

Although it is not an easy exercise, a Ricardian or Sraffian theory of international trade with traded intermediate goods is possible (Shiozawa 2007). It only requires a bit of unfamiliar mathematics: the theory of linear inequalities and the theory of convex cones. However, the economic principle is simple. In fact, the next theorem is obtained as follows:

Theorem 4.1 (Existence of the shared system of prices)

Let ε be an economy with M-country and N-commodities and a technology, which satisfies the following assumptions.

(1) Each country is equipped with a technology that comprises linear methods of production.
(2) Trade is conducted free of transport cost and without any legal restrictions.
(3) Labor is a unique primary factor and does not move across national boundaries.
(4) Capital goods are produced and traded across countries and can be used as inputs of production.

We further assume that

(5) each method of production is free of joint production (or the technology is a single product type)[3]*,*
(6) each country has a productive technique or a productive set of methods, and
(7) for each country the maximal rate of profit is higher than a fixed R.
Then, for any profit rate r less than or equal to R, there exists a set of wages $\mathbf{w}=(w^i)$ and a set of world prices $\mathbf{p}=(p^j)$, and they satisfy the following conditions:
(8) Each country has at least one competitive method of production in the sense that the profit rate of the method of production is equal to r, and
(9) for all methods of production, the profit rate is less than r.

A few definitions may be useful, for there are different ways for expressing the same object.

The basic concept in this paper is "method of production" which is called "technique" in Shiozawa (2007). The collection of all the methods of production is sometimes called technology of the economy. Each method represents a know-how, by which one can obtain, after a certain lapse of time, an output $\mathbf{b}=(b_i)$ in place of consuming inputs $\mathbf{a}=(a_i)$. The important assumption here is that the transformation of input into output is linear in the sense that when the transformation of $\mathbf{a}$ into $\mathbf{b}$ is possible, then the transformation of $\eta\mathbf{a}$ into $\eta\mathbf{b}$ is also possible for any non-negative number η. In this case, we say that these transformations are productions which belong to the method of production. To assume linearity of transformation is to assume constant returns to scale. When we have a set S of methods of production, the transformation of $\mathbf{a}$ into $\mathbf{b}$ is a

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[3] If the positive net product of a method of production includes more than two kinds of goods, the method is said to contain joint production. If the set of methods of production contains no method with joint production, the technology of the economy is said to be a single product type or a simple production type.

production belonging to the set S, if $(\mathbf{a}, \mathbf{b})$ is the sum of productions that belong to a method of production, or an element of the set S. The net product of the production $(\mathbf{a}, \mathbf{b})$ is $\mathbf{b}-\mathbf{a}$.

Another important assumption is assumption (5). A method is free of joint production, when the net output comprises a single type of good. The technology of the economy is a single product type, when any method of production is free of joint production. Shiozawa (2007) uses the term simple production economy. By this assumption, one can speak of an industry that produces a specific good. A method that produces goods j is called a method of industry j. There may be more than one method that produces the same types of goods or that belongs to the same industry. Thus, the choice of method of production is usually necessary.

However, this assumption is substantially restrictive, since it excludes the productions that employ durable capital goods such as machines and equipments. Theoretically, it is necessary to treat these capital goods as by-products, and thus, one cannot exclude joint production even when one works in an industry that produces one specific product. This assumption may then be a grave restriction while analyzing capital in a logical way. In reality, this restriction is practically lifted when durable capital goods keep their efficiency constant within the predetermined limit of durations. The examples of these calculations are provided in Sraffa (1960, section 82).

A bit of explanation for the difference in terminology will be useful. In Shiozawa (2007), the methods of production are called techniques. In this article, the word "technique" is used as a complete set of methods, that is a selection of the methods of production such that one method is chosen in each industry. In Shiozawa (2007), rather than technique, a word "system of techniques" is used, and the collection of all methods of production or technology is called "set of techniques". Otherwise, there are no major differences in terminology.

A technology is called productive when a net product that belongs to the technology is positive. A technique, or a set of methods, is productive when a net product belonging to the technique is positive. For each technique, a regular matrix A is associated. If a technique is productive, the associated matrix A is non-negatively invertible. This implies that A^{-1} exists, and all these entries are positive or zeros.

The theorem states that there are a system of wages $\mathbf{w}$ and a system of prices $\mathbf{p}$ by which each country has at least one worldly competitive method of production, and by consequence, a worldly competitive industry. Note that wages are given by a system of wages. Each country has a uniform level of wages, but they can be different for different countries. The essence of the theorem lies in the fact that one can find a good system of wages such that the minimum price theorem gives the requested systems of wages and prices. In addition, note that this theorem does not refer to the quantities of production and trade. These will be examined and analyzed when another theorem is obtained.

The above supposed situation is highly general. As the technology does not admit joint productions, fixed capitals are logically excluded. However, as I have mentioned above, they can be incorporated in the theory as long as we assume that the efficiency of the durable capital goods remains constant during the depreciation period. Each country may have different technologies. As a result, differences in climate and underground resources can be reflected in the differences of methods of production. For example, if a country enjoys a good climate for wheat production, the country has a method of wheat production with lesser coefficients than those of countries with less fortunate climate.

The above theorem is equivalent to Theorem 3.3 in Shiozawa (2007). The demonstration is simple when one uses F. E. Su's rental harmony theorem (Su, 1999). Two other versions of demonstrations are possible but they are exempt here. The existence of the so-called perfect specialization can also be proved when a generic condition is satisfied (Shiozawa 2007 Theorem 3.4). The vectors $\mathbf{w}$ and $\mathbf{p}$, which satisfy the above conditions (8) and (9), are called *shared*. Shared vectors form a closed cone of the positive orthant of dimension $M+N$, where M is the number of countries and N is the number of goods. Furthermore, we observe that this *shared cone* is endowed a mathematical structure called *fan*. If one analyzes this structure, one may determine all the possible patterns of specializations.

Another fundamental theorem is the next one. Here, I only give the theorem when the uniform growth rate g of the economy is 0. When the growth rate is positive, it is necessary to modify the theorem in such a way that the net product of a production should be redefined as $\mathbf{b}-(1+g)\,\mathbf{a}$. The duality between maximal frontier of productions and the shared cone of wage-price systems is valid only when the uniform profit rate is equal to the uniform growth rate. As this leads to an unnecessary complication of the formulae, I only state when the growth rate g and the profit rate r are 0. This does not imply that the new trade theory is constructed only when r and g are 0. This convention is made only for the simplicity of explanation.

In order to state the theorem, a few new symbols are necessary. Let $\mathcal{E}$ be an economy that satisfies the same assumptions as Theorem 4.1. Note that M is the number of countries and N is the number of commodities. We introduce a new symbol T, which is the total number of methods of production of all countries. (By consequence, T is at least equal to or greater than N times M). Let A be the coefficient matrix with T lines and N columns. By this definition A^{-1} has no meaning, since A is no more regular in general. Each line vector of matrix A is the net product of the production that belongs to each method of production and that uses one unit of labor as input. Let I be a matrix with T lines and M columns. The i-th entry of a line of I is 1 only when the line represents a method of production that belongs to the technology of country i. Finally, let $\mathbf{q}$ be the vector representing the amount of labor force of each country.

Theorem 4.2 (Maximal point of the production possibility set)

Let $\mathcal{E}$ be an economy that satisfies the same assumptions as Theorem 4.1. If $\mathbf{y}$ *is a maximal point of the production possibility set, then there exists a positive M-column vector* $\mathbf{w}$ *and positive N-column vector* $\mathbf{p}$ *such that*

$$A\mathbf{p} \leqq I\mathbf{w} \text{ and } \langle \mathbf{y}, \mathbf{p} \rangle = \langle \mathbf{q}, \mathbf{w} \rangle .$$

Conversely, if a couple of positive vectors $\mathbf{w}$, $\mathbf{p}$ *together with a net production vector* $\mathbf{y}$ *satisfy the above condition, then the net production vector* $\mathbf{y}$ *is one of the maximal points of the production possibility set.*

The shared cone, which may not be convex, is closely related to the production possibility set. For any pair of vectors $\mathbf{w}$ and $\mathbf{p}$ in the shared cone, we obtain a production possibility set for each country as follows:

If the set of all competitive techniques of a country m is given by $j(1)$, $j(2)$, ... , $j(k)$, then the set of competitively produced net products for the country m is given by

$$\left\{ \sum_{h=1}^{k} s_{j(h)} \mathbf{a}^{j(h)} \middle| \sum_{h=1}^{k} s_{j(h)} \leqq q_m \right\}.$$

Here, $\mathbf{a}^h$ is the net production coefficient vector, and q_m is the amount of labor force for country m, whereas s^h indicates the production scale for the technique h. Worldwide production is given by the Minkowski sum of the production sets for all countries. Any point thus obtained is proved to be maximal (or efficient), and they form a cell of the maximal production frontier. A beautiful dual correspondence theorem is obtained (Shiozawa 2007 Theorem 5.7), and we know that a facet (complex of dimension $N-1$) of the maximal production possibility set corresponds to a composing straight line of the shared cone. Those lines are of course finite, and the number of facets of the maximal frontier is finite.

An important fact here is that a facet covers a wide range of final demand compositions; however, the corresponding set of $\mathbf{w}$ and $\mathbf{p}$ are unique up to scalar multiplication. This can be called an extended non-substitution theorem for an international trade situation. As long as the global demand remains in the same facet, the relative wages and prices remain constant and quantity adjustment is necessary, in order that supplies are nearly equal to demands. Although the Samuelson's Limited Substitution Conjecture is false, as it will be shown in the next section, the fact shown above can be called "Limited Substitution Theorem," because wages and prices remain constant, and the set of competitive techniques remains the same (limited substitution) whenever the final demand changes within the same facet of the production frontier.

The Ricardo-Sraffa theory does not stop here. Only a special part of the entire theory is examined. The maximal production frontier provides a set of possible equilibrium states in which labor is fully employed in all countries and all industries work competitively. However, the main merits of the Ricardo-Sraffa theory remain on a different side of analysis. In contrast to the HOS theory, the new trade theory makes it possible to analyze out-of-equilibrium states.

As an example, consider net production $\mathbf{y}$, which is not in the set of maximal production frontier. We shall consider a positive system of wages $\mathbf{w}$ and a positive system of prices $\mathbf{p}$ that satisfy the next condition:

$$A\mathbf{p} \leqq I\mathbf{w}.$$

If $\mathbf{y}$ is a production that belongs to the technology of economy $\mathcal{E}$, then there exists a production activity vector $\mathbf{s} = (s_i)$ such that

$$\mathbf{y} = \mathbf{s}A \quad \text{and} \quad \mathbf{s}I \leqq \mathbf{q}.$$

Therefore, we obtain

$$\langle \mathbf{y}, \mathbf{p} \rangle = \langle \mathbf{s}A, \mathbf{p} \rangle = \langle \mathbf{s}, A\mathbf{p} \rangle \leqq \langle \mathbf{s}, I\mathbf{w} \rangle = \langle \mathbf{s}I, \mathbf{w} \rangle \leqq \langle \mathbf{q}, \mathbf{w} \rangle.$$

Two inequalities cannot be both an equality. In fact, if two equalities are obtained, by virtue of Theorem 4.2, $\mathbf{y}$ must lie in the maximal production frontier.

Then, what happens in economy $\mathcal{E}$? One of two inequalities

$$\langle \mathbf{s}, A\mathbf{p} \rangle < \langle \mathbf{s}, I\mathbf{w} \rangle \text{ or } \langle \mathbf{s}I, \mathbf{w} \rangle < \langle \mathbf{q}, \mathbf{w} \rangle$$

holds. If the first inequality holds, $s_t > 0$ for at least one method of production t with

$$\langle \mathbf{a}^t, \mathbf{p} \rangle > w_c,$$

where c is the country index of method t. This implies that in some industries, a method of production is operated even if it is not competitive. The state may produce a dual structure, or more generally, a differentiated structure inside the economy of a country.

If the second inequality holds, then for some countries c,

$$(\mathbf{s}I)_c < q_c.$$

This implies that there is unemployment in country c.

In sum, if net production $\mathbf{y}$ is not in the set of maximal production frontier, one of the next two conditions is violated:

(1) Labor is fully employed in all countries.

(2) All production is operated by means of competitive methods of production.

This state may often be observed, since there are many obstructions in order to arrive at a maximal point if one starts from any non-maximal point. One such state is autarchy. Obstructions may be the difficulties to find out an equilibrium state of world trade to finance the change of the production levels. The transition from the present state to an equilibrium state may require time to train workers appropriately. It is normal that one of these obstructions holds. Then, economy $\mathcal{E}$ must stay for a long time in a state where either (1) or (2) is violated. In this way, the out-of-equilibrium state of production and trade is analyzable in the Ricardo-Sraffa framework. However, the purpose of the present paper is not in this task. I stop here to return to the main flow of the topic.

5 Samuelson's conjecture is false.

In footnote 5 of the 2001 paper, Samuelson presented what he named Conjecture of Limited Substitution. It was stated as follows:

> No matter how many capital-using techniques occur in a 2-good, 2-country scenario involving fixed *localized* totals of population, generically (singular cases and joint-products aside), the maximal global production-possibility frontier will be concave and *two-faceted*.

The first part of the conjecture, which states that the frontier is concave, is true. The word "concave" is the short expression of "concave from the origin" and stands for the state that the production possibility set (composed of points maximal or not) is convex. However, the second part of the conjecture is not true as the following counter-example shows.

Each method of production is represented by a row vector. The first entry is labor input. The second and third entries indicate the net production of goods 1 and 2 for a unit scale of production. The scales are normalized so that the labor input is unit for a unit production. Let w^A and w^B be wage rates (measured by an international currency). The relevant ratio is relative wage rate w^A/w^B. We shall assume that $w^A=1$. Wage rate w^B takes any positive value.

A Counter-example to Samuelson's Conjecture

Countries	Output	Symbols of production method	Coefficients
Country A	good 1	A_{11}	(−1 11 0)
		A_{12}	(−1 14 −1)
		A_{13}	(−1 20 −4)
	good 2	A_2	(−1 −7 5)
Country B	good 1	B_1	(−1 6 −1)
	good 2	B_2	(−1 −1 4)

The switching points of patterns of specializations are given at $w^B=67/55$, $w^B=1$, $w^B=7/12$, and $w^B=23/72$. At these switching points, three methods become competitive. Between these switching points, only two methods are competitive. The list of competitive methods is given in Table 1.

The pattern of specializations is shared if and only if $67/55 \geqq w^B \geqq 23/72$. Figure 2 gives an idea of how the methods are related. Country A's methods are plotted by their second and third coordinates. Country B's methods will be represented by their second and third coordinates multiplied by $1/w^B$. Thus, Country B's methods move along a straight line when w^B changes. The line connecting the points represents the production possibility set, supported by the three methods. Key multipliers at the switching point are as follows: $55/67=0.82$, $1/1=1$, $12/7=1.71$, $72/23=3.13$.

The production possibility set is determined only when the amounts of labor for each country are given. We shall simply assume that $q_A=1$ and $q_B=1$. Then,

Table 1: List of Competitive Methods

The range of w^B when $w^A = 1$	The set of competitive methods
$w^B > 67/55$	A_{11} and A_2
$w^B = 67/55$	A_{11}, A_2, and B_2
$67/55 > w^B > 1$	A_{11} and B_2
$w^B = 1$	A_{11}, A_{12}, and B_2
$1 > w^B > 7/12$	A_{12} and B_2
$w^B = 7/12$	A_{12}, A_{13}, and B_2
$7/12 > w^B > 5/14$	A_{13} and B_2
$w^B = 23/72$	A_{13}, B_1, and B_2
$w^B < 23/72$	B_1 and B_2

the facets of the maximal frontier are given at each switching point.

When $w^B = 67/55$, methods A_{11}, A_2, and B_2 are competitive. Country B uses method B_2 and produces $(-1, 4)$. Country A uses methods A_{11} and A_2 and shares the labor such that the total labor input equals 1. When A_{11} uses all the labor force of the country, the production scale is equal to 1 and the net production is $(11, 0)$. When A_2 uses all the labor force, the net production is $(-7, 5)$. Thus, the production given by this point $w^A = 1$, $w^B = 67/55$ is the line segment that connects two points $(-7, 5)+(-1, 4)=(-8, 9)$ and $(11, 0)+(-1, 4)=(10, 4)$.

In a similar way, when $w^A = 1$, $w^B = 1$, methods A_{11}, A_{12}, and B_2 are competitive. Thus, the production frontier facet is the line segment, which connects $(11, 0)+(-1, 4)=(10, 4)$ and $(14, -1)+(-1, 4)=(13, 3)$. When $w^B = 7/12$, methods A_{12}, A_{13}, and B_2 are competitive. The frontier facet is the line segment that connects the points $(14, -1)+(-1, 4)=(13, 3)$ and $(20, -4)+(-1, 4)=(19, 0)$. When $w^B = 23/72$, methods A_{13}, B_1, and B_2 are competitive. Country A produces using only the method A_{13}. The net product is $(20, -4)$. Country B shares labor force in two productions using methods B_1 and B_2. Thus, the line segment is given by the two points $(20, -4)+(-1, 4)=(19, 0)$ and $(20, -4)+(6, -1)=(26, -5)$. All of these results give the total maximal frontier as it is given in Figure 3.

The above example satisfies the assumptions of Samuelson's Conjecture. In fact, it is a two-country two-good case, and the example is generic in the sense that even if the coefficients are changed within a sufficiently small amount, the number of facets remains four. Even if we confine frontier only in the positive quadrant, the number of facets is more than three. This is clearly a counter-example for his Conjecture.

Samuelson adds a few words for the situation when the number of countries is increased. He introduces Ronald Jones's remark: In a generic three-country (probably two-goods) case, the plausible conjecture will be that the frontier is concave three-faceted. The above example provides a counter-example to Jones's

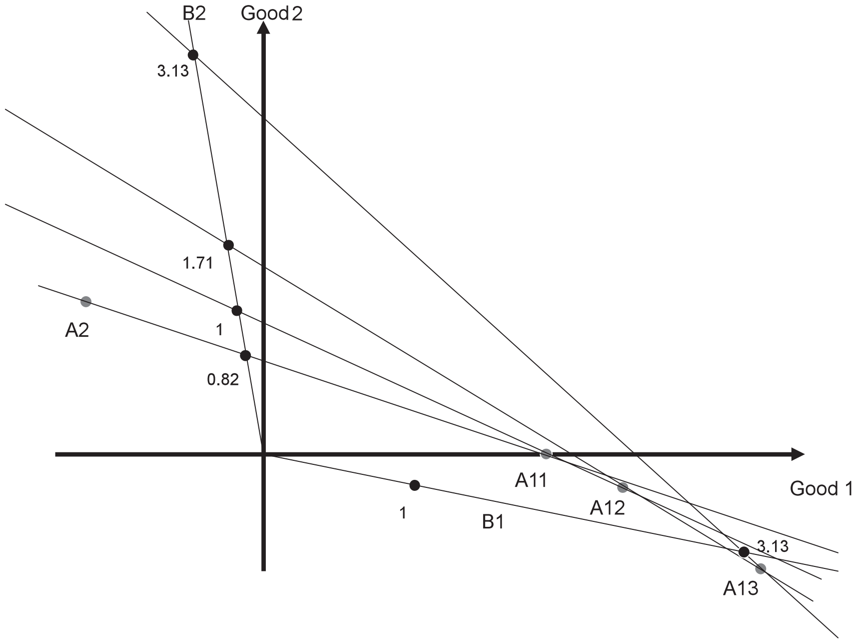

Figure 2: A counter-example for the Samuelson's conjecture
NB. Scales of coordinates are different. The scale of the y- coordinate is 2 times the scale of the x-coordinate.

conjecture about what happens when a third country is added. It is sufficient to take the set of methods for country C $(-1, 1, 0)$ and $(-1, 0, 1)$. It is evident that for any pair w^A and w^B, there exists at least one competitive method for country C. The profit for a unit production for each method is $p^1 - w^C$ and $p^2 - w^C$. When p^2 is bigger than p^1, the method that produces good 2 is competitive. When p^1 is less than p^2, the method $(-1, 0, 1)$ is competitive, whereas method $(-1, 1, 0)$ gives a negative profit. On the other hand, when $23/72 < w^B < 67/55$, p^2 is always bigger than p^1. Thus, it is always method $(-1, 0, 1)$ that remains competitive. When the labor force of country C is 1, its production is always $(0, 1)$. The maximal frontier is thus shifted by a unit upward, and the form of the maximal frontier remains the same. However, in this case, the number of facets is four, even if we count those facets that lie in the positive quadrant. Jones's conjecture is also false.

In Appendix 2 of his paper, Samuelson reviews the "trade generalization of 1960 Sraffa." He explains the two-country two-good case, and at the end of Appendix 2 he remarks: "My described techniques can be modified to handle ... cases of more than two goods and more than two countries."

This remark is questionable, since the many-good many-country case requires a new formulation essentially different from the two-good case or the two-country

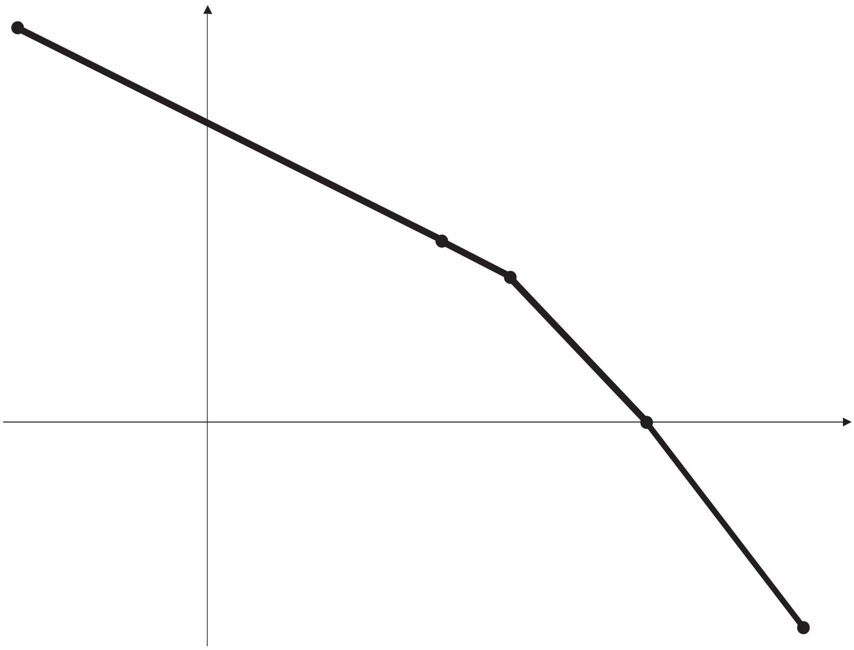

Figure 3: Maximal frontier has 4 facets.

case. For example, in the two-country case, one can appeal to the middle-value theorem, that is, one can find a shared vector by moving wages of a country continuously from a sufficiently small to a sufficiently large number. The many-country case requires the existence theorem for a shared pattern of specializations (See Method 1 in the proof of Theorem 3.3 of my paper).

Another point of difference lies in the necessity to analyze the structure of the shared cone, which is closely related to the patterns of specializations. The dual correspondence between two modal decompositions (one of the wage simplex and another of the production frontier) is essential to understand that many-good many-country case, as explained in Shiozawa (2007 Section 5). Samuelson gives no indication of these ideas. Even for the two-country two-good case, if we know the dual correspondence between modal decompositions, it is easy to guess the general form of the maximal frontier, and consequently, the maximal number of facets. Samuelson disregards all these important points of analysis, and it is difficult to believe that he has really constructed a Sraffa-type general trade theory for the many-country many-good case.

6 An episode as a guise of conclusion

As Samuelson's example shows, the Sraffian bonus is considerably large. Samuelson's implicit criticism against Sraffa and the Sraffians was correct. By its theoretical and actual importance, a theory of input trade is necessary. Theoretically, as Sraffa's 1960 book reveals, the circular nature of industrial production is essential. Its actual importance is apparent because the absolute volume and relative weight of input trade are increasing. Furthermore, processing trade is becoming more and more important[4]. Processing trade normally takes the form of triangle trade. China imports key devices from Japan, assembles them, and exports the products to the United States. Here, the usually assumed rest-of-the-world story is no longer valid. We must construct a general theory of input trade for a many-country many-good case. The first step toward this general theory has been taken by Shiozawa (2007). This is a reply to Samuelson's criticism, which is now invalid.

The extensions of the Ricardian trade theory to the input trade case have an unexpected effect on policy discussions, as witnessed by the recent debate between famous economists like Greg Mankiw and Dani Rodrik[5]. The debate started with the post of Daniel Drezner. Drezner objected to Rodrik's saying that Rodrik elides the biggest gain from trade—lower prices. Rodrik responded that (1) trade may decrease the prices of imported goods but will increase those for export goods and (2) gains from trade for the consumers depend on the relative weight of imported and exported goods. Mankiw defended the idea that trade can lower prices. By pointing out that people treat nominal wages as the numeraire, he claimed that all prices are lowered after trade when they are compared to closed state prices, and the real wage rate will be increased. Rodrik did not admit Mankiw's contention and claimed that the Stolper-Samuelson theorem denies that the real wages of workers rise as a result of free trade. Finally, Mankiw posted a short response with the title "Ricardo vs. Heckscher-Ohlin." Mankiw argues in this post that he relies on the Ricardian trade theory instead of the more common HOS theory and explains how this is derived. Further, he contended that the Ricardian trade theory is more realistic than the HOS theory, as the latter assumes that capital cannot move from country to country. However, he continues, in today's global economy, capital is highly mobile across national borders and concludes that in light of this fact, in his opinion, the Ricardian theory of trade is more useful than the HOS theory.

As a student of Ricardian trade theory, I am of the same opinion as Mankiw. However, it is noteworthy that Mankiw's contention contains a logical slip. The traditional Ricardian theory does not assume capital in any explicit form, and the trade of capital is not taken into consideration. Mankiw's contention is right only when a new trade theory of the Ricardo-Sraffa type is constructed. The construction of a Ricardo-type general theory of input trade is thus relevant to today's highly academic discussions, both in theory and policy.

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[4] Fragmentation is but a special form of processing trade.
[5] The entire series of interventions is summarized by Mark Thoma in the post of April 28, 2007 in his blog *The Economist's View*.

References

Aspromourgos, Tony. (2004) "Sraffian Research Programmes and Unorthodox Economics." *Review of Political Economy*, **16**(2): pp. 179–206.

Baldone, Salvatore, Fabio Sdogati, Lucia Tajoli. (2007) "On Some Effects of International Fragmentation of Production on Comparative Advantages, Trade Flows and the Income of Countries." *The World Economy*, **30**(11): pp. 1726–1769.

Chipman, John S. (1965) "A Survey of the Theory of International Trade: Part 1, The Classical Theory." *Econometrica*, **33**(3): pp. 477–519.

Chipman, John S. (1971) "International Trade with Capital Mobility: A Substitution Theorem," In Jagdish Bagwati, Ronald W. Jones, Robert Mundell, and Jaroslav Vanek, (eds.) *Trade, Balance of Payments, and Growth*, Amsterdam: North-Holland, pp. 201–237.

Deardorff, Alan V. (2005) "Ricardian Comparative Advantage with Intermediate Inputs." *The North American Journal of Economics and Finance*, **16**(1): pp. 11–34.

Giammanco, Maria D. (1998) "Scarcity of Resources in a Sraffian Framework of International Trade." *Metroeconomica*, **49**: pp. 300–318.

Gontijo, Claudio. (2000) "The Ricardian Theory of International Trade: A Criticism." Text Para Discussan No. 141, Universidade Federal de Minas Gerais.

Jones, Ronald W. (1961) "Comparative Advantage and the Theory of Tariffs: A Multi-Country, Multi-Commodity Model." *Review of Economic Studies*, **28**(3): pp. 161–175.

Jones, Ronald W. (1980) "Comparative Advantage and Absolute Advantage." *Schweizerische ZeitSchrifts für Vorlkswirtsschaft and Statistik* (*The Swiss Journal of Economics and Statistics*), **116**: pp. 235–259.

Jones, Ronald W. (1994) "Trade with Capital Mobility: A Ricardian Approach." *Nagasaki Prefectural University Review*, **28**(1): 77–89.

Jones, Ronald W. (2000) *Globalization and the Theory of Input Trade*, Cambridge, MA: MIT Press.

Knell, Mark and Martin Srholec. (2006) "Fragmentation, Heterogeneity and Intra-product Trade within Europe." Draft of 12 September 2006.

Maneschi, Andrea. (1992) "Ricardo's International Trade Theory: Beyond the Comparative Cost Example." *Cambridge Journal of Economics*, **16**(4): pp. 421–437.

Mankiw, Gregory. (2007) "Ricardo vs Heckscher-Ohlin." Post of April 28, 2007 of the Greg Mankiw's Blog. http://gregmankiw.blogspot.com/2007/04/ricardo-vs-heckscher-ohlin.html

Marioris, Theodore. (2004) "Pure Joint Production and International Trade: a Note." *Cambridge Journal of Economics*, **28**(3): pp. 449–456.

McKenzie, Lionel W. (1954) "Specialization and Efficiency in World Production," *The Review of Economic Studies*, **21**(3): pp. 165–180.

McKenzie, Lionel W. (1956) "Specialization in Production and the Poduction Possibility

Locus." *The Review of Economic Studies*, **23**(3): No. 62, 1955–56. pp. 56–64.
Parinellom, Sergio. (1970) "Introduzione ad una teoria del commercio internazionale." *Studi Economici*, **25**: pp. 267–321.
Pasinetti, Luigi L. (2005) "The Sraffa-enigma: Introduction." *The European Journal of the History of Economic Thought*, **12**(3): pp. 373–378.
Pasinetti, Luigi L. (2006) "Paul Samuelson and Piero Sraffa — Two prodigious minds at the opposite poles." in Szenberg, Michael, Lall Ramrattan and Aron A. Gottesman (eds.) *Samuelsonian Economics and the Twenty-first Century*, Oxford University Press, pp. 146–464.
Reiter, Stanley. (1953) "Trade Barriers in Activity Analysis." *The Review of Economic Studies*, **20**: pp. 174–180.
Sanyal, Kalyan, and Ronald W. Jones. (1982) "The Theory of Trade in Middle Products." *American Economic Review*, (March) pp. 16–31.
Samuelson, Paul. (1966) "A Summing Up." *Quarterly Journal of Economics*, **80**(4): pp. 568–583.
Samuelson, Paul. (2001) "A Ricardo-Sraffa Paradigm Comparing Gains from Trade in Inputs and Finished Goods." *Journal of Economic Literature*, **39**(4): pp. 1204–1214.
Samuelson, Paul. (2004) "Where Ricardo and Mill Rebut and Confirm Arguments of Mainstream Economists Supporting Globalization." *Journal of Economic Perspectives*, **18**(3): pp. 135–146.
Shiozawa, Yoshinori. (2007) "A New Construction of Ricardian Trade Theory — A Many-country, many-commodity Case with Intermediate Goods and Choice of Techniques." *Evolutionary and Institutional Economics Review*, **3**(2): pp. 141–187. http://www.jstage.jst.go.jp/article/eier/3/2/3_141/_article.
Sraffa, Piero. (1960) *Production of Commodities by Means of Commodities*, Cambridge University Press.
Steedman, Ian. (ed.) (1979a) *Fundamental Issues in Trade Theory*, London: MacMillan.
Steedman, Ian. (1979b) *Trade Amongst Growing Economies*. Cambridge: Cambridge University Press.
Steedman, Ian. (1999) "Production of Commodities by Means of Commodities and the Open Economy." *Metroeconomica*, **50**(3): pp. 260–276.
Steedman, Ian. (2002) "Marx after Sraffa and the Open Economy." A paper prepared for the 4th Annual Conference, Association for Heterodox Economics.
Steedman, Ian and Metcalfe, John S. (1981) "On Duality and Basic Commodities in an Open Economy." *Australian Economic Papers*, **20**: pp. 133–141.
Su, Francis Edward. (1999) "Rental Harmony: Sperner's Lemma in Fair Division," *American Mathematical Monthly*, **106**: pp. 930–942. Available at http://www.math.hmc.edu/~su/papers.dir/rent/pdf
Subasat, Turan. (2003) "What Does the Heckscher-Ohlin Model Contribute to International Trade Theory? A Critical Assessment." *Review of Radical Political Economics*, **35**(2): pp. 148–165.
Takamasu, Akira. (1986) "On the Production Possibility Frontier in a Sraffa-Leontief Economy." *The Manchester School of Economies and Social Studies*, **54**(2): pp.

202–207.

Thoma, Mark. (2007) "April 28, 2007 On the Other Hand ... Rodrik versus Mankiw (Others Also Weigh In)" *Economist's View* (Blog) http://economistsview.typepad.com/economistsview/2007/04/on_the_other_ha./html

Vasudevan, Ramaa. (2004) "Comparative Advantage, Terms of Trade and the Neo-Ricardian Contribution to Trade Theory," http://www.umass.edu/ peri/ pdfs/ NSVasudevan.pdf See html version in Google.