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ARTICLE

A New Construction of Ricardian Trade Theory—A Many-country, Many-commodity Case with Intermediate Goods and Choice of Production Techniques—

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Abstract

Ricardian trade theory is one of the most famous theories of economics but appears to have been little developed. Many attempts were made to extend the theory to multi-country, multi-commodity cases, but none succeeded to construct a general theory that included intermediate goods. A need to include intermediate goods within the theory was evident, but hurdles to introduce intermediate inputs were high. Intermediate goods change the entire structure of analysis; when they are traded, the price of a good is dependent of the prices of imported inputs. Consequently, prices should be determined simultaneously for prices of all countries. The present paper has succeeded in overcoming these difficulties and describes how wages, prices and productions are related. It analyzes the M-country, N-commodity case with choice of techniques and trade of intermediate goods in general terms, thus presenting a new basis for international trade theory. New light was shed on topics like gains and losses from trade, international wage rate discrepancies, and price and quantity adjustments. On a theoretical plane, the new construction eliminates a traditional weakness of the Ricardian theory. The traditional Ricardian theory acknowledged labor as the only input and excluded capital in any form. The new theory, presented here, analyzes capital goods as traded intermediate inputs.

1. Introduction

Ricardian trade theory is one of the most famous theories of classical economics. It is cited in many introductory course textbooks on international trade. The logic of comparative advantage is even now impressive and profound. A famous nuclear physicist Stanislav Ulam once asked Paul Samuelson if there is any non-trivial theorem in economics. Many years later Samuelson found a good answer: the theory of comparative advantage is one.

Nearly two centuries have passed after the publication of Ricardo's Principles (1817).

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In spite of its evident importance and the claims of wide applicability[1)], the theory appears to have been little developed. In the 19th century J. S. Mill argued about mutual demand. In the first half of the 20th century F. D. Graham examined a more general case than Ricardo's 2-country, 2-commodity case, but the analysis was always based on numerical examples. Observations were generalized to the 2-country, many-commodity case by Harberler (1930). Dornbusch, Fischer and Samuelson (1977) extended the analysis to the case of continuum of commodities. The many-country, 2-commodity case was examined by Vainer (1937). In 1950's and in 1960's some general analysis for *M*-country *N*-commodity case appeared. McKenzie (1954b), for example showed the existence of equilibrium for Graham's model of international trade. Jones (1961) discovered a formula that could tell the possible pattern of specialization. After that period no substantial improvement appeared in this field[2)].

This does not mean that all the important problems were solved. On the contrary, the models so far analyzed had two crucial defects. (1) Inputs were restricted to labor as a unique factor and no material inputs were admitted. This implied that intermediate goods were excluded from any theoretical analysis of international trade. (2) Choice of techniques was not admitted. This is what is necessary when one wants to analyze technical change and development.

Intermediate goods change the entire structure of analysis. If intermediate goods were not traded, the prices of any products of a country can be determined inside that country. Thus all prices move proportionally to the wage rate of the country. When intermediates are traded, price of a good is dependent of the prices of inputs and those inputs may be imported from foreign countries. Consequently, prices should be determined simultaneously for all countries. The price of a commodity depends not only on the wage rate of the producing country but may well be dependent on other countries' wage rates. The price effects of wage rate changes become very complicated and autarchy prices of each country have little connections with international prices. The choice of techniques adds further complication to this situation.

Hurdles to introduce intermediate inputs were high. The very notion of comparative advantage was not clear. As Deardorff (2005a, 2005b) reported, many competing definitions were proposed without any conclusive arguments. Some examinations based

1) See for example Suranovic (1997–2004) Section 40-1 "The Theory of Comparative Advantage-Overview."

2) A unique exception might be Takamasu, 1991, pp. 34–41, in which Takamau examined the cases with intermediate goods.

on numerical examples were made but only sporadically (See Amano 1966, Higashida 2005, and others). More than twenty years ago, I myself tried unsuccessfully to extend the Ricardian model to a general case. My paper of 1985 dealt only with the case of two countries. I occasionally returned to this problem, and recently found a way to overcome the difficulties that occur with the introduction of intermediate goods. This paper deals with the many-country, many-commodity case, with traded intermediate goods, and choice of techniques in general terms. The results were first written in Japanese, but this article is the first publication in English[3].

The importance of intermediate goods is apparent. They compose an important part of world trade. Recent discussions on fragmentations, outsourcings and intra-industry trade are all related to intermediate goods. It is relatively unusual to discuss detailed models without any general theory of intermediate goods. The standard Heckscher-Ohlin theory is not well fitted to analyze intra-industry trade. Krugman and others presented explanations from increasing returns to scale, but most of the theory was limited to a special setting such as two-country and three- or four-commodity cases. The lack of a general theory is apparent. Those topics are not explicitly examined in this paper but the theory presented in this paper may well serve as such a general theory.

The gains from trade, when intermediates are traded, have a different dimension from those when final products alone are traded. An elegant example was recently provided by Paul A. Samuelson (2001). Figure 1 is an illustration depicting to what extent the trade of intermediate goods enlarges the world production possibility set.

The main themes of this paper are set out as follows. Section 2 is preparatory; necessary concepts and assumptions are explained. Section 3 is the most complex part of the present paper. Subsection 3-1 argues for the existence of a world minimal production price vector when the wage rate for each country is given. This is only an application of minimal price theorem for a closed economy but it presents the starting point of all further discussions of this paper. In subsection 3-2, competitiveness of a technique and other related definitions are presented and the existence theorem of a shared pattern of specialization is specified. In subsection 3-3, the notion of modal decompositions is given for the wage rate delta. In subsection 3-4, a proof of a more difficult existence theorem is presented. Finally in subsection 3-5, a result of numerical experimentation is given.

[3] See Shiozawa (2007). Although this paper and that of Shiozawa (2007) have many common discussions, they are written as independent papers and in several places the logical constructions are different.

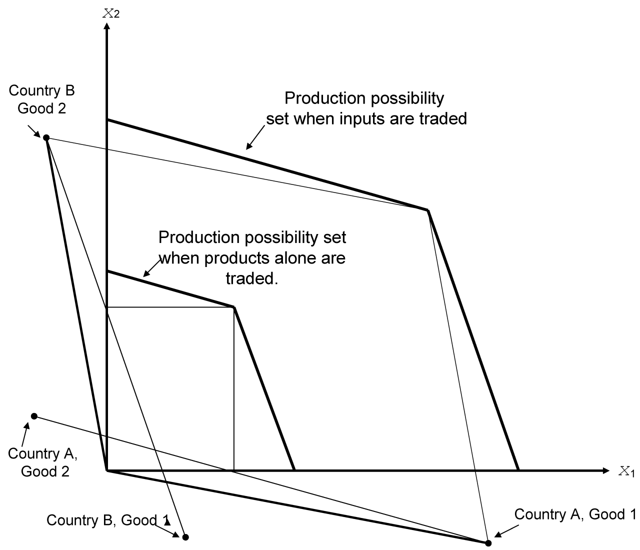

Fig. 1. Two Kinds of Gains from Trade

Section 4 discusses the gains from trade. Typical gains from trade are explained in subsection 4.1, but in subsection 4.2 the possibility of unemployment and other losses from trade is also discussed.

Section 5 discusses the forms of the production possibility set. After an introduction to some theorems on production possibility points and on maximal points, subsection 5.3 presents an important dual correspondence theorem between two modal decompositions. Subsection 5-4 presents some conclusions that are easily derived from the dual correspondence theorem.

Section 6 discusses the famous controversy between Ricardo and Graham on one side and Mill and others on the other. The conventional focal point was what determines the prices: are supply-side conditions sufficient to determine prices, or do demand conditions intervene in the determination? A new light is shed on this old debate. Real conflict was not between supply and demand but between price adjustment and quantity adjustment. According to the view of dual correspondence theorem, I argue that quantity adjustment is predominant to price adjustment.

Finally, in section 7, the questions of international wage differentials are approached. It is obvious that there are large discrepancies in wages between countries. International trade theory should explain why these discrepancies continue in the presence of (nearly free) trade. This is done here, because price determination also implies wage rate determination. Thus, policy implications are clear. It is necessary for poor countries to change their set of techniques[4]. This may be the most important message of this paper.

The message of the Hechsher-Ohlin model is distorted. If the factor price equation theorem is applicable, then the wage rates should be equalized for all countries. This is totally contradictory to what is actually observed.

For a considerable time the Heckscher-Ohlin model was regarded as the standard model of modern trade theory. Discordances in this model with empirical evidence have, on many occasions, been reported. In addition, it has also some serious theoretical defects. Capital is treated as a factor and it is presumed that factors are not traded. However, capital goods such as raw materials, parts of machines and durable capital goods are commonly traded internationally. Not only they are traded, as mentioned above, they actually represent an important part of world trade. Moreover, the concept of capital as a mass, independent of prices, cannot be defended. It may be time to replace Hechsher-Ohlin theory with extended Ricardian theory.

In the following discussion, the main mathematical tool is real linear algebra or the theory of linear inequalities. Elementary knowledge of convex polytopes and topology is useful. The mathematics employed here is not particularly common in recent economics but all necessary knowledge is classical and obtainable either in Koopmans (1951), in Gale (1960) or in Nikaido (1961).

2. Main Scheme of the Theory

The present paper presents a new method for a construction of Ricardian trade theory. By this method, we can include in the analysis all of the following situation:

(a) M-country N-commodity case.

(b) The choice of techniques.

(c) Intermediate goods (produced and traded).

A system of techniques is a set of techniques, in which each product has a technique that produces it. A production belongs to a system of techniques, when it is a sum of productions and each production belongs to a technique of the system. A system of techniques is by definition productive when a net product of the production belonging to the system is positive for all goods. We assume three conditions for production techniques of each country:

(1) Linear production techniques.

[4] Among conditions that determine this set of techniques social technologies, including social institutions, are as important as physical or material technologies

(2) Simple production-type techniques.

(3) Existence of a productive system.

In the case of single country this assumption is called the Leontief-Sraffa model. This is merely an international version of the Leontief-Sraffa model, which is also a faithful generalization of the Ricardian trade model. Some additional explanations are needed for each condition.

(1) The production technique is linear or maintains constant returns to scale. Each production of a technique is expressed as $\alpha(\mathbf{a}, \mathbf{b})$, where α is a positive real number, $\mathbf{a}$ is an input vector and $\mathbf{b}$ an output vector[5)]. All productions are proportional as far as productions belong to the same technique.

(2) The production techniques are simple in the sense that net product consists of only a single kind of goods as far as production belongs to a technique[6)]. With this assumption any production technique has two kinds of membership: a country and an industry. A technique is said to belong to industry j when the net product consists of goods j. Functions $C(\ )$ and $G(\ )$ are defined in such a way that we have equations $C(\tau)=h$ and $G(\tau)=j$, when the technique τ belongs to the country h and belongs to industry j.

(3) Each country has at least one productive technique system. In some cases, it is sufficient to assume that the world as a whole has at least one productive technique system.

When a system of techniques γ is given, the matrix $A(\gamma)$ is usually arranged so as to have a technique that produces j-th commodity in j-th line. The elements of matrix $A(\gamma)$ are positive for all diagonal entries and non-positive (i.e. zero or negative) for all off-diagonal entries. When the system is productive, $A(\gamma)$ is invertible and $A(\gamma)^{-1}$ is non-negative.

The techniques of each country are assumed to be different, reflecting the differences in each country's technological knowledge, climate and geography, and resource allocations. The set of all techniques is denoted by Ξ. Labor power is presumed to be the

5) The present paper deals only with zero profit case. Examination of the positive profit cases is also possible if one takes as output vector the discounted output vector $(1/(1+r))\mathbf{b}$. In this case, production possibility set should be interpreted as the set of net surplus product in the growing economy with growth rate $1+r$.

6) Joint production of the type of electric salt decomposition that produces sodium and chloride at the same time is excluded. Durable capital goods can be treated as long as they retain constant efficiency within a prefixed lifetime. For the simplicity of the explanations we assume that production techniques are all simple in the sense defined above. See Shiozawa (1975).

unique primary factor, which is fixed to each country and does not move internationally. Although there is constant immigration between countries, cultural differences are usually large enough to limit the proportion of immigrants to a small part of the population. Most countries have also a quota system or other measures to limit immigration into their countries. So the immobile work force assumption remains relevant for the contemporary world. The labor power of a country i is denoted q_i. We set $\mathbf{q}=(q_i)$. The set of techniques Ξ and distribution of labor powers $\mathbf{q}$ are two fundamental conditions that determine all possible states of the economy and are assumed to be fixed[7)].

The choice of techniques is an essential part of international trade theory. Under assumption (3), each country has a technique that produces each good. Then if there are M countries in the world, there are M techniques that produce the same good. One should determine which technique of which country is competitive and which are not. This is the question of specialization. The determination of pattern of specialization is nothing other than the choice of techniques in worldwide competition. The existence of plural techniques in a single country for each good makes only a small difference for the analysis.

3. Main Process of Analysis of Wage Rates and Prices

3.1 Minimal price vector that corresponds to the wage-rate vector

The key point of the new construction is to start with a column vector $\mathbf{w}=(w^h)$. Here, w^h is the wage rate of country h, given in terms of a common international currency. We call $\mathbf{w}$ a wage rate vector. When there are M countries in the world, the vector is of dimension M.

The starting with wage rates is inevitable, because any wage-rate vector has a corresponding minimum production-price vector $\mathbf{p}=(p^j)$, whereas it is exceptional for a price vector to have a wage-rate vector that generates the price vector $\mathbf{p}$ as the minimum price vector. The linear product of a line vector $\mathbf{x}$ and a column vector $\mathbf{v}$ of the same dimension is denoted by $\langle \mathbf{x}, \mathbf{v} \rangle$.

The main theorem used here is the minimal price theorem.

Theorem 3.1 (Minimal price theorem)

Suppose a country where techniques are linear and simple, and which has a productive system of techniques. Then there exists a system of techniques γ^* that gives a minimal

7) The effects of changing Ξ and $\mathbf{q}$ are important targets of analysis but they are beyond the reach of this paper.

price vector $\mathbf{p}=(p^j)$. *This means that for any normalized vector of production coefficients, the profit inequality*

$$w \geq \langle \mathbf{a}^\tau, \mathbf{p} \rangle$$

holds for any technique τ *and wage rate* $\mathbf{w}$. *In particular, if the technique belongs to system* τ^**, the above condition holds with equality. If we pick up techniques that belong to* τ^* *and arrange the vectors in a matrix form and write it* $A(\tau^*)$*, the price vector can be expressed as*

$$\mathbf{p}=wA(\gamma)^{-1}\mathbf{1},$$

where $\mathbf{1}$ *is a column vector all composed of* 1.

This theorem is sometimes called non-substitution theorem. A different expression of the theorem is

$$\mathbf{p}(\gamma^*) \leq \mathbf{p}(\gamma)$$

for any productive γ, when we write $\mathbf{p}(\gamma)$ the minimal-price vector for the system of techniques γ.

This theorem can easily be extended to the case of international trade if the positive rate-wage vector $\mathbf{w}=(w^1, w^2, \ldots, w^M)$ is given. When the wage-rate vector and a price vector are given, the excess profit net of capital cost for a technique τ is

$$\langle \mathbf{a}^\tau, \mathbf{p} \rangle - w^{C(\tau)},$$

where $C(\tau)$ is the country index of the technique.

Theorem 3.2 (Minimal price theorem for the world)

Suppose that all the assumptions of Section 2 are satisfied and that a positive wage-rate vector $\mathbf{w}=(w^1, w^2, \ldots, w^M)$ *is given. Then there is a system of techniques that gives the minimal production price* $\mathbf{p}=(p^1, p^2, \ldots, p^N)$ *for the world* (*the minimal price theorem*). *This means that wage vector* $\mathbf{w}$ *and price vector* $\mathbf{p}$ *satisfy the following two propositions*:

(I) *For any technique, the excess profit net of capital cost is 0 or negative.*

(II) *For any industry, there is at least one technique whose excess profit net of capital cost is equal to* 0.

In the following discussion, we simply say "excess profit" instead of saying "excess profit net of capital cost", because excess profit is always regarded as net of capital cost.

The above condition (I) can be expressed in a matrix form

$$A\mathbf{p} \leq I\mathbf{w}\,, \tag{1}$$

if A and I are defined as follows. A is an L-line N-column matrix composed of i-th line $\mathbf{a}^{\tau}$ where the technique τ is numbered i. The i-th line of I is a vector whose elements are 0 for all columns except h, and 1 for the h-th column when the technique τ belongs to the h-th country.

Condition (II) cannot be expressed in such a simple way. However, we will later find an equivalent condition so that conditions (I) and (II) hold at the same time.

When wage-rate vector and price vector satisfy inequality (1), they correspond to the pairing of wage-rate and price vectors and are briefly called w-p vector in this paper. Excess profit of a technique, when it is numbered i, is the i-th line of the expression $I\mathbf{w} - A\mathbf{p}$.

3.2 Competitive techniques for a wage-rate vector

For any wage-rate vector $\mathbf{w}$, one can calculate the minimum production-price vector $\mathbf{p}$. Take a corresponding pairing ($\mathbf{w}$, $\mathbf{p}$) or a w-p vector and fix them for a while. The excess profit net of capital cost is calculated for each technique. Then we can classify competitive techniques and non-competitive ones.

Definition 3.1 (Competitive techniques)
Let a w-p vector ($\mathbf{w}$, $\mathbf{p}$) *be given. A technique is called competitive when the excess profit of the technique is zero. Otherwise a technique is non-competitive. Two or more techniques of the same industry, of the same country or of different countries, can be competitive. For any wage vector* $\mathbf{w}$ *a set of competitive techniques* $CT(\mathbf{w})$ *is assigned.*

Definition 3.2 (Pattern of specialization)
A pattern of specialization is a subset of the set of all techniques. The set of patterns of specialization is the power set $\mathfrak{P}(\Xi)$ (*or the set of all subsets*) *of the set of techniques* Ξ. *Each wage-rate vector* $\mathbf{w}$ *induces a pattern of specialization which is no other than the set of competitive techniques* $CT(\mathbf{w})$.

Note that a set of competitive techniques does not necessarily contain techniques of all countries. When a pattern of specialization contains for each country at least one technique of that country, the pattern is called a “shared” pattern of specialization. When a wage-rate vector $\mathbf{w}$ generates a shared pattern of specialization, the vector is said to be “sharing”. The same adjective is applicable for a w-p vector ($\mathbf{w}$, $\mathbf{p}$), when it generates a shared pattern of specialization.

Consider a situation where each country produces something without making a loss at a given w-p vector. "Without making loss" means here that the production of each country is a sum of productions belonging to competitive techniques. Then each country should have at least one competitive technique. At that situation the set of competitive techniques must be shared.

Sometimes this sharing wage-rate vector is called the "equilibrium rate", but the present paper does not use this term, because the concept is defined independently from the state of supply and demand and remains useful even out with the typical equilibrium point.

The first problem arises whether there is a wage-rate vector that induces a shared pattern of specialization. Fortunately, this existence is assured for any economy of Section 2. This actually leads to the next theorem.

Theorem 3.3 (Existence of shared pattern of specialization)
Consider the situation given at the beginning of Section 2. Let Ξ *be a set of techniques that satisfies assumptions* (1), (2) *and* (3) *of that section. Then there exists a positive wage-rate vector that induces a shared pattern of specialization.*

(Proof of Theorem 3.3)
There are various methods to prove theorem 3.3. Let us cite three of them.

Method 1. (Combinatorial geometry)
The theorem is equivalent to F. E. Su's rental harmony theorem. See Su (1999). It is easy to show these conditions of the theorem starting from the assumptions (1), (2) and (3).

Method 2. (Modal decomposition, weak version)
With a slight revision of the proof of strong existence theorem, which will be given later in this section, one obtains a proof of the existence of wage-rate vectors that induce the shared pattern of specialization. In fact, instead of taking $sCC(P)$ for each polytope P of the cell decomposition, take $CT(P)$ and let $wCC(P)$ be the collection of country numbers that appear in $CT(P)$. Since the technique of a country is at least competitive for a given $\mathbf{w}$, $wCC(P)$ is not an empty set. Therefore, if there is no participating wage-rate vector, one can construct a similar continuous function from Δ to $\Delta/G(\Delta)$, which leads to a contradiction.

Method 3. (Geometry of the production possibility set)
Suppose each country, is given a positive quantity of work force. Let $\mathbf{q}=(q_i)$ be a line vector, where q_i is the work force of country i. The proposition 5.4 attests that there

exists a maximal point **y** of the production possibility set P in any positive direction. Therefore, theorem 5.2 provides a w-p vector (**w**, **p**) when it satisfies the conditions $A\mathbf{p} \leq I\mathbf{w}$ and $\langle \mathbf{y}, \mathbf{p} \rangle = \langle \mathbf{q}, \mathbf{w} \rangle$. Proposition 5.3 asserts the existence of an activity vector s which satisfies equations $\mathbf{y}=\mathbf{s}A$ and $\mathbf{q}=\mathbf{s}I$. The latter equation means that some technique is active for each country and one can deduce that each country has at least one competitive technique. ⋈

3.3 Modal decomposition of wage rate Δ

Strong interest has been paid for a particular pattern of specialization. That situation is usually called complete specialization. However, this notion is sometimes ambiguous. So in the present paper different terminology is used.

Definition 3.3 (Strongly competitive country)
Let a list of competitive techniques be given. If for an industry j any country other than country i has no competitive techniques of the industry j, then i is called strongly competitive country for the industry j.

Definition 3.4 (Strongly-shared pattern of specialization)
For any wage-rate vector **w***, take an paring* (**w**, **p**) *which satisfies the inequality* (1). *When the pattern of specialization induced by the vector* (**w**, **p**) *satisfies the following two conditions*:

(1) *Every country has at least one competitive technique that belongs to that country.*

(2) *Any competitive technique of the same industry belongs to the same country,*

the wage-rate vector **w** (*or the paring* (**w**, **p**)) *is said to induce strongly-shared pattern of specialization*.

Note that in the definition of the concept strongly-shared pattern of specialization, nothing is stipulated if there is another competitive technique of country i and of industry j. Country i may or may not have two or more competitive techniques in industry j. The next theorem gives a significantly stronger result than theorem 3.3, but does not hold for all situations.

Theorem 3.4 (Existence of strongly-shared patterns of specialization)
Let the economy be that of section 2. Let the number of commodities N be at least equal to the number of countries M. Techniques are assumed to satisfy conditions (1), (2) *and* (3) *of that section. Suppose, in addition, that for any non-negative wage-rate vector there is at least one industry for which one country is strongly competitive. Then, there exists*

an M-dimensional convex cone, in the interior of which the wage-rate vector induces a strongly-shared pattern of specialization[8].

Theorem 3.4 is proved with the aid of modal decomposition. Modal decomposition is also a necessary tool for the understanding of the dual structure which will be explained in section 5.

Let us first explain the notion of modal decomposition of wage-rate vectors. This can be defined both on the standard simplex Δ of wage-rate space or on a non-negative octant. The definition on the simplex is more visible and easier to imagine[9]. The standard simplex Δ of dimension $M-1$ is defined by the formula

$$\Delta=\{(w^1,\dots,w^M)\mid w^1+\cdots+w^M=1, w^1\geq 0,\dots,w^M\geq 0\}\,.$$

For any vector $\mathbf{w}$ on the simplex Δ, the set of competitive techniques $CT(\mathbf{w})$ is defined. This is a map from Δ to the set of all possible patterns of specialization $\mathfrak{P}(\Xi)$. This map can be in a sense invertible.

Take any pattern of specialization *Ptn*. For any pattern of specialization *Ptn*, a subset $R(Ptn)$ of the simplex Δ is defined by the following formula:

$$R(Ptn)=\{\mathbf{w}\mid Ptn\subset CT(\mathbf{w})\}\,.$$

This is equivalent to say that $R(Ptn)$ is the set of vectors $\mathbf{w}=(w^1,\dots, w^M)$ which satisfy the conditions:

$$A\mathbf{p}\leq I\mathbf{w} \quad \text{and} \quad A(Ptn)\mathbf{p}=I(Ptn)\mathbf{w}\,. \tag{2}$$

Here, $A(Ptn)$ and $I(Ptn)$ are the extracted matrices of A and I for lines belonging to *Ptn*.

By consequence, $R(Ptn)$ is a convex polytope as the set of solutions of a system of linear inequalities. Note that inequalities in (2) are all accompanied with equalities. This implies that $R(Ptn)$ is closed. The closed convex polytope $R(Ptn)$ is called *modal cell* corresponding to the pattern of specialization *Ptn*. It may be the empty set, when the system of inequalities (2) has no solutions.

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8) In the case where no trade is admitted for intermediates, Jones (1961) presented a necessary and sufficient condition for the existence of strongly-shared pattern of specialization, although in that publication he only argued about necessity and no proof of the sufficiency was given. As Higashida (2005) found it, when intermediate goods are admitted in the trade, there exist cases where there are two or more strongly-shared patterns of specialization.

9) Modal decomposition can be defined one the set of non-negative price vectors. But that modal decomposition is much more skewed one. It is only defined on some part of the price simplex. See Corollary 5.8.

The collection D of all modal cells which correspond to a pattern of specialization has the following properties:

1. All elements of D are convex polytopes.
2. An empty set is an element of D.
3. When a polytope P is an element of D, any face of the polytope P is also an element of D.
4. If two polytopes P and Q are elements of D, the common set of P and Q is also an element of D.
5. Every point of Δ belongs to an element of D.

A collection D of subsets of a set S, which satisfies five conditions above (when Δ is replaced by S for condition 5), is called cell decomposition of the set S. Thus, the collection D of all modal cells is a cell decomposition of the simplex Δ. This cell decomposition is called *modal decomposition* of the wage rate Δ.

A similar decomposition is possible for a non-negative octant (of high dimension) of wage-rate space, if one replaces polytopes by convex cones when it is necessary to designate that the modal decomposition is related to technology.

If a modal cell P corresponds to a pattern of specialization Ptn, then any point of the modal cell induce a pattern of specialization which is a superset of Ptn. This does not imply that the points of a modal cell have the same pattern of specialization. However, if the cells are restricted to their relative interior, the correspondence is much more simple.

The relative interior of a polytope P is the set of interior points of P as points of the smallest affine space that includes P[10]. The relative interior of the set composed of only a point is the point itself, or more exactly the set itself. As for modal cells, the relative interior of the modal cell $R(Ptn)$ can be expressed as the set of solutions of the following system of inequalities:

$$A(\Xi/Ptn)\mathbf{p} < I(\Xi/Ptn)\mathbf{w} \quad \text{and} \quad A(Ptn)\mathbf{p} = I(Ptn)\mathbf{w}\,,$$

where Ξ/Ptn is the complementary set of Ptn in Ξ. $A(S)$ and $I(S)$ are as before the extracted matrices of A and I for lines belonging to the set S. Any two points in the relative interior of a modal cell have the same pattern of specialization. There is a one-to-one (but not onto) correspondence between the modal decomposition and the set of all patterns of specialization.

The relative interior of a polytope P is denoted by rel.int(P). With this notation,

[10] An affine space is a subset of a vector space that is obtained by displacing a subspace by a parallel transportation.

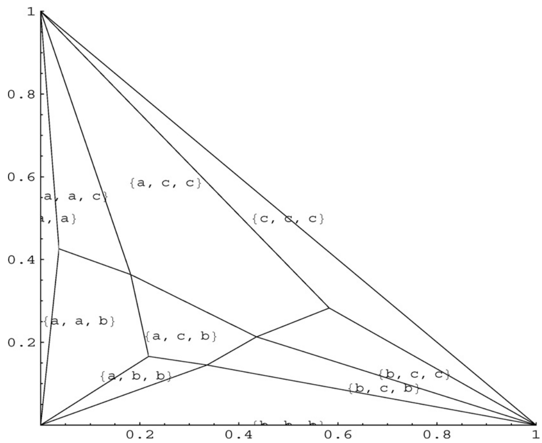

Fig. 2. Modal Decomposition of Wage Delta.

relations between patterns of specialization and modal cells take a simple expression. In fact, for any possible pattern of specialization, one gets the equality

$$CT^{-1}(Ptn) = \text{rel.int}(R(Ptn)) .$$

If one starts from a modal cell P, the same relations can be expressed as follows:

$$Ptn = CT(\text{rel.int}(P)) \quad \text{and} \quad R(Ptn) = P .$$

When the number of countries is 3, Δ is a triangle in a plane and can be expressed on a sheet of paper. One example of such modal decompositions is given in Figure 2.

The proof of theorem 3.4 requires a lengthy preparation. Before attacking this task, let us examine how general is the supposition additionally imposed in the theorem. Note that all $\mathbf{w}$ vectors, which are proportional with each other, induce the same set of competitive techniques. It is then sufficient to consider all things on a normalized vector.

Suppose that there exists a point $\mathbf{w}$ that does not satisfy the assumed condition. Take the minimal price vector $\mathbf{p} = \mathbf{p}(\mathbf{w})$ associated to the wage-rate vector $\mathbf{w}$. Then there are for every industry j at least two competitive techniques belonging to different countries. For each of these techniques τ_1 and τ_2, equations

$$w^{C(\tau_k)} = \langle \mathbf{a}^{\tau_k}, \mathbf{p}(\mathbf{w}) \rangle$$

hold for $k=1$ and 2. At any point $\mathbf{w}$, $\mathbf{p}(\mathbf{w})$ is expressed by a system of techniques γ in such a way that

$$\mathbf{p}(\mathbf{w}) = A(\gamma)^{-1} I(\gamma)\mathbf{w}\,.$$

Note that the number of combinations that give a system of techniques is finite. If j_1, j_2 are two country numbers of the techniques, this means

$$\mathbf{w}^{j_1} - \mathbf{w}^{j_2} = \langle \mathbf{a}^{\tau_1} - \mathrm{a}^{\tau_2}, C(\gamma)\mathbf{w} \rangle\,,$$

where $C(\gamma) = A(\gamma)^{-1} I(\gamma)$ is a (M, N) matrix of coefficients. The vector $\mathbf{w}$ which satisfies this equation, lies, in general, in a hyperplane of co-dimension 1 in the standard simplex Δ. The N of such hyperplanes do not meet, in general, at a point in common when $N \geq M$, for Δ is of dimension $M-1$. These are requirements that cannot hold, in general, for an arbitrary given set of techniques of an economy. The expression "in general" means that by changing one or more coefficient of techniques the proposition is satisfied. Changing the combination γ does not change the situation in terms of generality. One can always choose points that satisfy a finite number of requirements in the neighborhood of the present coefficients[11].

In subsection 3.5 we will estimate explicitly the probability for an economy to satisfy the existence theorem.

3.4 Existence of strongly-shared patterns of specialization (Proof of theorem)

In this subsection a proof of theorem 3.4 is presented. It proceeds in six steps.

Step 1

The set of techniques satisfies conditions (1), (2) and (3) of section 2. Number of commodities N is supposed to be equal to or greater than number of countries M. Suppose, in addition, that for any wage-rate vector there is at least one industry for which a country is strongly competitive. For any element P of the modal decomposition D, choose a point $\mathbf{w}$ in the relative interior of the polytope P and take the set of competitive techniques. As we have seen above, this set of competitive techniques is uniquely determined. Then the set of strongly competitive countries for some industry is uniquely defined and denoted by $sCC(P)$. From the assumption of the theorem, $sCC(P)$ is non-empty.

[11] The expression "in general" is usually ambiguous and often therefore criticized. In the present argument, generality means that exceptional cases lie only in a set of measure 0 if one introduces an appropriate measure in the space of coefficients. This interpretation is affirmed by the result of subsection 3.5.

Step 2

For any convex polytope P, the barycenter of P is denoted as $G(P)$. When a list of country-valued function $sCC(P)$ is given, a continuous function f from Δ to Δ it is defined by the following procedure.

For any barycenter of an element P of modal decomposition D, $f_B(G(P))$ is defined by the formula

$$f_B(G(P)) = \frac{1}{\#(sCC(P))} \sum_{j \in sCC(P)} \mathbf{e}(j)\,.$$

Here $\#(S)$ for a set S stands for the number of elements of S. The vector $\mathbf{e}(j)$ is a vertex of Δ, which is expressed as $(\delta(i,j))$ $i=1,2,\ldots,M$ for Kronecker's delta $\delta(i,j)$[12]. The devisor $\#(sCC(P))$ is necessary only to keep $f_B(G(P))$ on Δ. The function f_B is well defined over the set of all barycenters. The following steps extend this function f_B inductively:

1. Take elements of D of dimension 0 or 0-faces. These are polytopes whose base set consists of a single point. On the base set of 0-faces, the function f is defined by

$$f(X) = f_B(X)\,.$$

2. Suppose that function f is defined on the base set of all faces of D whose dimension is less than d. Take a polytope P among the d-dimensional elements of D. If Q is a facet of P, the barycenter $G(P)$ is outside Q. Consider a conic polytope R with vertex $G(P)$ and with base Q. Any point X of R is on a line segment that connects $G(P)$ and a point Y of facet Q. When X is expressed as $X=aG(P)+(1-a)Y$, then function f is defined on X by

$$f(X)=af(G(P))+(1-a)f(Y)$$

As function f is defined at $G(P)$ and at any point Y of Q, this is well defined.

3. Do the same extensions of function f for any facet Q' of polytope P. When two facets Q and Q' have a common set P' function f is already defined on P' and two definitions coincide on the points of the line that connects $G(P)$ and a point in P'. Thus the definition can be extended on all points of polytope P of dimension d.

4. A function f is defined over all points of Δ when the definitions arrives at dimension $M-1$. The obtained function is peace-wise linear and continuous.

[12] Kronecker's δ is defined as $\delta(i,j)=1$ for $i=j$ and $\delta(i,j)=0$ otherwise.

Step 3

The theorem can be demonstrated by contradiction. Suppose there are no elements of modal decomposition D that induce strongly-shared pattern of specialization. This means that, for any polytope P of D, $sCC(P)$ cannot be equal to the set of all country numbers. Then, the above constructed function f: $\Delta \rightarrow \Delta$ never takes the value $G(\Delta)$.

In fact, if there is no element of D that induces strongly-shared pattern of specialization, there is a country m that is not a member of $sCC(P)$ for any element of D. Then, for any polytope P of D, $f(P)$ is included in a facet of Δ. More explicitly, if m is the number not included in $sCC(P)$, $f(P) \subset \Delta(m)$ where $\Delta(m)$ is the facet that does not include $\mathbf{e}(m)$. This proposition can be proved by induction on dimension d for the elements of D.

For $d=0$, the proposition means that $f(G(P))$ is included in a facet of $\Delta(m)$. This is verified easily by examining the inductive definition of the step 2. Suppose then that the proposition is verified for all elements of D whose dimension is less than d. For an element P of D of dimension d let m be a number that is not a member of $sCC(P)$. Then, the base set of P is included in the facet $\Delta(m)$. $G(P)$ and any facet Q of P, are subsets of P and in this title $sCC(G(P))$ and $sCC(Q)$ do not contain m as their member. From this assumption of the induction $f(G(P))$ and $f(Q)$ are included in $\Delta(m)$. The inductive definition (2 of the step 2) suggests that $f(P)$ is itself a subset of $\Delta(m)$. So the proposition is asserted and $f(\Delta)$ does not contain $G(\Delta)$.

Step 4

From step 3, if there is no element of D that induces strongly-shared pattern of specialization, then one gets a continuous function f: $\Delta \rightarrow \Delta$ that does not take the value at $G(\Delta)$. The function $f \mid \partial \Delta$, which is the restricted function of f on the boundary $\partial \Delta$, is homotopic to the identity function from S^{M-2} to S^{M-2}. However, the existence of a continuous function f: $\Delta \rightarrow \Delta / G(\Delta)$ means that the identity function over S^{M-2} is homotopic to a constant function. This contradiction follows because we have assumed that there is no element that induces strongly-shared pattern of specialization. Therefore, it is assured that there exists at least one element that induces strongly-shared pattern of specialization.

Step 5

Let a polytope P of the modal decomposition D induce a strongly-shared pattern of specialization. At a point $\mathbf{w}$ of P, the wage-rate vector $\mathbf{w}$ generates a minimal price vector p. Given a w-p vector $(\mathbf{w}, \mathbf{p})$, any country j has at least one technique, τ, that is competitive and no other country has a competitive technique of the same industry. This is an open condition in the sense that if **w** moves in a neighborhood of the original **w**, the conditions remain valid. So if P induces a strongly-shared pattern of specialization, P must include the open subset. This means on Δ that P is of dimension $M-1$.

Step 6

All propositions on Δ and its convex subset can be translated into propositions on the non-negative octant and convex cones with vertex 0. The result obtained in step 5 asserts that there exists an M-dimensional convex cone, in the interior of which the wage-rate vector induces a strongly-shared pattern of specialization. This completes the proof of the theorem. ⋈

In the case of an economy that does not admit trade of intermediate goods, there is only one polytope that induces a strongly-shared pattern of specialization and such a pattern of specialization is unique. In the general case, however, where intermediate goods are traded, an economy might have two or more polytopes that induce strongly-shared pattern of specialization and the patterns of strongly-shared pattern of specialization may not be unique.

3.5 Numerical experiments on the occurrence of non-existent cases

Theorem 3.3 holds for all cases given in section 2, whereas theorem 3.4 holds only for an economy that satisfies the additional condition. As examined above, this condition must hold *in general*. This means that if coefficients are randomly chosen from a range of certain intervals the probability of the cases in which the condition is not satisfied should be zero. With the aid of computers, we can estimate this probability. The only hurdle to surpass is to code the necessary program.

Table 1 shows some results of numerical experiments. The result given on the third line confirms the prediction that the non-existent case probability is zero. In cases of integral coefficients the probability may not be zero (the first line). However, if the moving range of coefficients increases, the probability to find a non-existent case becomes more difficult and approaches the real coefficient setting.

4. Gains from Trade and Origins of Trade Conflicts

What type of gains do trading countries obtain? What type of sufferings should they endure, if any harm to trade could exist at all? These are the subjects with which trade theories have been preoccupied since the very beginning of economics. In most cases, theorists assured the gains from trade but were very reluctant to admit any harm to trade.

Table 1. A result of numerical experiments

Dimensions & fields	Range of diagonal elements	Range of non-diagonal elements	Number of cases examined	Non-existent cases	Probability
3 Integer	[1, 9]	[0, 5]	100,000	487	0.0049
3 Integer	[1, 18]	[0, 10]	100,000	0	0
3 Real	[1, 9]	[0, 5]	100,000	0*	0
4 Integer	[1, 9]	[0, 5]	100,000	77	0.0008

(Experiments by Mathematica)

* In the course of this examination, it is necessary to check if the inverse of a matrix is non-negative. In the real coefficients case, the computation is approximate. The matrices that contain negative components whose absolute value is less than 10^{-6} were classified as non-negative.

However, in the real world, trade often raised conflicts between countries. It is not rare that one hears claims that industries of one country are suffering due to the rapid increase in exports from other countries. Some people express these situations as an export of unemployment.

The majority of economists customarily rebuff these claims by professing that there is no real trade conflict. In total, trade is beneficial to both countries and complaints are based on misunderstandings, they suggest. They have no malicious or political intentions. They believe what the theory tells to them. They only adopt a standard tool of economic analysis and apply them to international trade and conclude that there are gains from trade for both trading countries.

The standard framework of economic analysis is equilibrium analysis and economists do not regard that the very adoption of this method has a strong influence on the problem. In equilibrium, everything goes well. No harm occurs. However, this is only a consequence of the equilibrium framework[13]. In order to analyze trade conflict it is necessary to analyze what happens outside the equilibrium. The method of the present article enables this analysis and thus enlarges the scope of trade theories beyond that of neoclassical tradition, which is strongly dependent on equilibrium analysis.

In the analysis of gains and losses, it is necessary to look at the situation from the standpoint of each interest group. In subsection 4.1 the gains for the country as a whole are examined. In subsection 4.2 the same situation will be examined from the viewpoint

[13] Even a very strong analyst such as Krugman (1996) has some tendency to think and reason in this way.

of each interest group.

4.1 Gains from trade for the nation

To see what kind of gains we get from trade, suppose for each country an autarky $E(j)$: country j is producing net product $\mathbf{y}(j)$, using labor power $q(j)$. If $\mathbf{s}(j)$ is the activity vector of country j then

$$\mathbf{y}(j)=\mathbf{s}(j)A(j) \quad \text{and} \quad q(j)=\mathbf{s}(j)I(j)$$

where $A(j)$ is the net production coefficient matrix for techniques of country j and $I(j)$ is a column vector of $L(j)$ lines all composed of 1.

Suppose now trade begins. The next theorem follows.

Theorem 4.1 (Trade with constant demand)

Suppose an economy of M-country N-goods with a choice of techniques and in which intermediates goods are used in production and are tradable internationally. Suppose that a wage-rate vector $\mathbf{w}=(w^i)$ *and its associated price vector p induce a shared pattern of specialization, then there exists a worldwide production that satisfies the following 5 conditions*:

1. *The worldwide net production is equal to the sum of the net productions of each country's autarky.*
2. *Each country consumes the same amount of commodities as the autarky.*
3. *All goods that are traded internationally are produced by competitive techniques with regard to w-p vectors* $(\mathbf{w}, \mathbf{p})$.
4. *In terms of total values, each country exports as much as it imports.*
5. *The total labor time of each country, is less than or equal to the labor time of each country's autarky case. If activities in one country, include an activity that is not competitive with regard to vectors* $(\mathbf{w}, \mathbf{p})$*, then the total sum of labor time of the world is less than the sum of labor time of each country's autarky.*

Note that in the new state, internationally traded commodities are exchanged at the same price vector $\mathbf{p}$. Prices of other commodities are not specified. They may be traded with prices between old autarky prices and international prices. So the situation in the theorem assumes a double structure: international prices for international trade and domestic prices for domestic trade and productions.

In the proof of this theorem, we use the next lemma.

Lemma 4.2

Let B be an $L\times N$ *matrix with any line vector* $\mathbf{b}^h$ *that has only one positive element and* *all other elements are either zero or negative. Suppose in addition each column has at least one positive element. If there exists a positive column vector* $\mathbf{p}$ *which satisfies*

$$B\mathbf{p}=0$$

or $\langle \mathbf{d}^h, \mathbf{p}\rangle=0$ *for all line vectors* $\mathbf{d}^h$ *of D, then there is a non-zero non-negative vector* $\mathbf{u}=(u_h)$ *such that*

$$\sum_h u_h \mathbf{b}^h = 0\,.$$

(Proof of lemma 4.2)

When B is a square matrix, this is a known lemma for a simple production type matrix. Let A be the matrix of all off-diagonal components of B with signs inverted, in which the notion of decomposability and indecomposability of A is well known. We say B is decomposable or indecomposable if A is decomposable or indecomposable, respectively. When the matrix B is indecomposable, the lemma holds with positive vector $\mathbf{u}$. When B is decomposable, there is a positive $\mathbf{u}$ for the fundamental indecomposable part, and $\mathbf{u}=0$ for the remaining part. The proof of this part is omitted for the brevity of the paper. If matrix B is not square, take a square submatrix of B with the same properties. The lemma gives a non-negative non-zero vector $\mathbf{u}$ and the augmented vector s with zeros for omitted lines satisfies the lemma for the original matrix. ⋈

(Proof of theorem 4.1)

In this proof, i, j and h denote country number, commodity number and technique number, respectively. Let $\mathbf{y}(i)=\sum_h \mathbf{s}^{\natural}(i)_h \mathbf{a}^h$ be the country i's production vectors of the autarky. We adopt the same notations A and I as above[14]. Let vectors $(\mathbf{w}, \mathbf{p})$ induce a shared pattern of specializations. Then an inequality $A\mathbf{p} \leq I\mathbf{w}$ holds. Each technique that holds with equality is competitive and each technique that holds with strict inequality is non-competitive.

To prove this theorem, it is necessary and sufficient to show the existence of production vectors $\mathbf{z}(i)=\sum_h \mathbf{t}(i)_h \mathbf{a}^h$ that satisfy the following conditions:

(1) $\mathbf{t}(i)_h \geq \mathbf{0}$ for all i.

(2) $\sum_i \mathbf{z}(i) = \sum_i \mathbf{y}(i)$.

(3) If $\mathbf{t}(i)_h > \mathbf{s}^{\natural}(i)_h$, then h is competitive.

(4) $\langle \mathbf{z}(i) - \mathbf{y}(i), \mathbf{p}\rangle = 0$ for all i.

[14] See subsection 3.1, equation (1).

(5) $\sum_h \mathbf{t}(i)_h \leq \sum_h \mathbf{s}^{\natural}(i)_h$.

(6) For some country i (5) holds with strict inequality.

If we use expressions $\mathbf{s}(i) = (s(i)_h)$ and the likes, vectors $\mathbf{y}(i)$ and $\mathbf{z}(i)$ are expressed as $\mathbf{s}(i)A$ and $\mathbf{t}(i)A$. Let $\mathbf{q} = (q_i)$ where q_i is the maximum labor time available in country i. Without loss of generality, we suppose

$$\sum_h s(i)_h = q_i .$$

The production possibility set of the country i is denoted by $P(i, q_i)$. This means that

$$\mathbf{z}(i) = \sum_h t(i)_h \mathbf{a}^h \in P(i, q_i) \Leftrightarrow \sum t(i)_h \leq q_i$$

with the assumption that $t(i)_h$=0 for all h that do not belong to i. Vectors $\mathbf{z}(i)$ will be found in the following procedure.

Vectors $\mathbf{w}$ and $\mathbf{p}$ satisfy the inequality $A\mathbf{p} \leq I\mathbf{w}$ for all lines. Equality holds for lines that are related to competitive techniques and strict inequality for lines that are related to non-competitive techniques. Extracting each country's part of matrix A and I, one gets an inequality

$$A(i)\mathbf{p} \leq w^i I(i) .$$

Here $I(i)$ is a column vector with all entries 1. Set

$$A(i)\mathbf{p} = w^i J(i) ,$$

then elements of $J(i)$ are equal to 1 for competitive techniques and smaller than 1 for non-competitive techniques.

Consider a set $E(i)$ of production activities $\mathbf{s}(i)$ belonging to country i which satisfy

$$\mathbf{s}(j)J(i) = \sum_{C(h)=i} s(i)_h J_h(i) = q_i .$$

Then for any activities $\mathbf{r}(i)$ in $E(i)$, the net products

$$\mathbf{x}(i) = \sum_h r_h(i)\mathbf{a}^h(i)$$

satisfy the equality

$$\langle \mathbf{x}(i), \mathbf{p} \rangle = \langle \mathbf{y}(i), \mathbf{p} \rangle ,$$

resulting in two chains of equalities:

$$\langle \mathbf{x}(i), \mathbf{p} \rangle = \sum_h r_h \langle \mathbf{a}^h(i), \mathbf{p} \rangle = w^i r(i) J = w^i q_i ,$$

$$\langle \mathbf{y}(i), \mathbf{p} \rangle = \sum_h s_h \langle \mathbf{a}^h(i), \mathbf{p} \rangle = w^i s(h) J = w^i q_i .$$

Let $S = \{S_1, S_2, \ldots, S_M\}$ be a collection of sets whose components are numbers of competitive techniques of the country i and for any element h of set S_i let

$$\mathbf{b}(h) = q_i \mathbf{a}^h - \mathbf{y}(i) .$$

Note that $\mathbf{y}(i)$ is the net production vector in autarky, and so $\mathbf{y}(i)$ is non-negative. The vector $\mathbf{a}^h$ has only one positive component and all other components are zeros or negative. Therefore, $\mathbf{b}(h)$ is a simple production type. Moreover,

$$\langle \mathbf{b}(h), \mathbf{p} \rangle = q_i \langle \mathbf{a}^h, \mathbf{p} \rangle - \langle \mathbf{y}(i), \mathbf{p} \rangle = q_i w^i - q_i w^i = 0 .$$

Then, the set of vectors $\mathbf{b}(h)$ forms a matrix B that satisfies the conditions of lemma 4.2 and there exists a line vector $\mathbf{u} = (u_h)$ such that $\mathbf{u}B = 0$.

Take a maximal real number η which satisfies for all i the conditions

$$\mathbf{y}(i) + \eta \sum u_h \mathbf{b}(h) \in P(i, q_i) . \tag{3}$$

Here h in the summation runs for all competitive h with $C(h) = i$. Let

$$\mathbf{x}(i) = \eta \sum u_h \mathbf{b}(h) .$$

with the same summation range. Then, the production switches from $\mathbf{y}(i)$ to $\mathbf{z}(i) = \mathbf{x}(i) + \mathbf{y}(i)$ satisfy the conditions of theorem 4.1, when the consumption of country i remains $\mathbf{y}(i)$ and the difference $\mathbf{x}(i)$ of net productions and consumptions are traded internationally.

Then in fact,

(1) $\sum_i \mathbf{y}(i) = \sum_i \mathbf{z}(i)$,

(2) $\mathbf{y}(i) + \mathbf{x}(i) = \mathbf{z}(i)$ for all i.

(3) positive components of $\mathbf{x}(i)$ all belong to competitive techniques.

(4) $\langle \mathbf{x}(i), \mathbf{p} \rangle = 0$ for all i.

(5) By definition $\mathbf{z}(i) = \mathbf{y}(i) + \mathbf{x}(i)$ is included in $P(i, q_i)$ and inclusion (3) means that total labor input of $\mathbf{z}(i)$ is equal to or smaller than q_i. Moreover, if positive u_h exists for

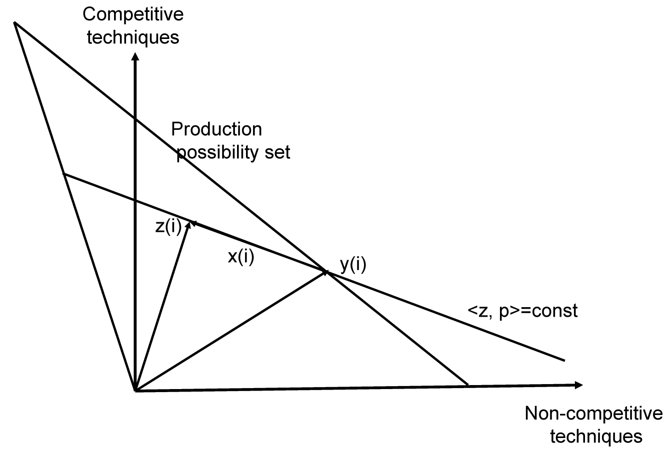

Fig. 3. The same world production with reduced labor input.

country i, inclusion (3) and the relation $J(h) < I(h)$ for non-competitive h implies the total input of labor is smaller than q_i. See Figure 3 for clarification. ⋈

Theorem 4.1 asserts that countries can generally obtain gains from trade: the same consumption with less labor. However, the interests are different with different social groups. We will examine this in the next subsection.

4.2 Views from different interest groups

Suppose countries start to trade after autarky. The situation assumed in theorem 4.1 is easily attainable, if sharing wage rates are found. People then transfer activities from non-competitive productions to competitive productions since demand levels are already known. Thus, they can easily find the necessary scales of production according to existing demand.

The gains from trade are visible for workers who continue to be employed. They get wages at the same rate as before and in a short time prices begin to approach international prices. If international prices prevail in a domestic economy, it is evident that

$$\frac{1}{w^i}\mathbf{p} \leq \frac{1}{w^i}\mathbf{p}(i)$$

where $\mathbf{p}$ is the international price vector and $\mathbf{p}(i)$ is the price of country i when it runs in autarky. In terms of commodities, real wage increases with increased trade if the strict inequality $\mathbf{p}^h < \mathbf{p}^h(i)$ holds. Even if the prices do not move entirely to $\mathbf{p}$, employed workers will get some portion of the benefit.

One problem is that the new situation requires a smaller amount of labor. If each worker continues to work the same hours per week as before, then some of the workers should lose work. Thus, in this model, unemployment occurs. People who lose their work cannot enjoy cheaper prices, if it is a consequence of trade. By the start of trade, it is possible that the first effects of trade are the emergence of unemployment. This is one of the losses due to trade.

For the industrialists also, the effects are different. If they work in a competitive industry with competitive techniques, they have a chance to expand activities and to acquire more profits. If they work in a non-competitive industry, using non-competitive techniques, they are obliged to decrease levels of their activity in this industry. Price are reduced and salable volume decreases and they loose some of their profits.

Of course, as shall be described in the following section, a full employment state does exist. In the long run, the economy will probably succeed to approach to such a situation and everybody will be happier in this state than in the case of autarky. The question is how rapid economy can get to this happy situation. If it is a long process that requires trial and error, people suffer from trade at least for a while.

5. Production Possibility Set

It is useful to know the shape of the production possibility set. Analysis of the production possibility set is in one sense simpler than that of wages and prices. The assumptions imposed on investigated economies are the same as before: there are M countries, N commodities, and L techniques of simple production type belonging to a country. Moreover, the following two conditions are assumed: (1) All the production techniques are simple and each country has for any commodity one technique that produces the commodity. (2) Each country has a set of techniques that are productive.

Assume each country has a positive amount of labor-power q_i. Labor is a unique primary factor and all other goods can be produced by a combination of a certain quantity of labor and a set of appropriate amount of various goods. Outputs are traded and consumed either as consumer consumption or as input to a production in the next period. In an economy with the set of techniques $\Xi = \{\Xi(i)\}$, a net production is possible when it is a non-negative combination of techniques $\mathbf{a}^h$, i.e $\sum_h s_h \mathbf{a}^h$ for non-negative s_i and labor input for each country is equal to or less than q_i. The last condition can be written as

$$\sum_{h \in \Xi(i)} s_h \leq q_i \quad \text{for any } i.$$

Let an $L \times N$ matrix A be the collection of all techniques $\mathbf{a}^h$ and a $L \times M$ matrix I, the collection of vectors of the unit type, meaning that the entries at (h, i) are 1 if they represent techniques of number h of the country i and 0 for all other entries. Then, a possible net production $\mathbf{y}$ and the labor input y_0 of the production are given by the formula

$$\mathbf{y} = \mathbf{s}A \quad \text{and} \quad y_0 = \mathbf{s}I$$

for some non-negative L-vector $\mathbf{s}$. In the following discussion production is considered only as those possible productions and the adjective possible is omitted.

The production possibility set of the economy is denoted $P(\mathbf{q})$. It can be expressed

$$P(\mathbf{q}) = \{\mathbf{y} \in R^N \mid \mathbf{y} = \mathbf{s}A,\ \mathbf{s}I \leq \mathbf{q},\ \mathbf{s} \geq \mathbf{0},\ \mathbf{s} \in R^L\}$$

where $\mathbf{q}$ is labor-power vector composed of each country's labor-power q_i. As the set of points satisfying a system of linear inequalities, the production possibility set is a convex polytope.

5.1 Basic characteristics of production possibility set

The next theorem gives a necessary and sufficient condition that a point of R^N is an element of the production possibility set.

Theorem 5.1 (Production possibility set)

For a point $\mathbf{y}$ *of* R^N *to be a point of the production possibility set* $P(\mathbf{q})$*, it is necessary and saffiicient that any pairing of column vectors* $\mathbf{w}$ *and* $\mathbf{p}$ *of dimension M and N which satisfies*:

$$A\mathbf{p} \leq I\mathbf{w} \quad \textit{and} \quad \mathbf{w} \geq 0$$

satisfies the inequality

$$\langle \mathbf{y}, \mathbf{p} \rangle \leq \langle \mathbf{q}, \mathbf{w} \rangle.$$

(Proof of 5.1)

If $\mathbf{y}$ is a point of $P(\mathbf{q})$, there exists by definition a non-negative activity vector $\mathbf{s}$ such that

$$\mathbf{y} = \mathbf{s}A \quad \text{and} \quad \mathbf{s}I \leq \mathbf{q}.$$

The last conditions are equivalent to the existence of non-negative $\mathbf{t}$ such that

$$\mathbf{y} = \mathbf{s}A \quad \text{and} \quad \mathbf{s}I + \mathbf{t} = \mathbf{q}\,.$$

Take a $(L+M)\times(N+M)$ matrix

$$\begin{pmatrix} A & -I \\ O & -E \end{pmatrix},$$

where O is an $M\times N$ matrix of elements zeros, and E is an identity matrix of dimension M. Then, the existence of the above **s** and **t** is the existence of a solution

$$(\mathbf{s},\mathbf{t})\begin{pmatrix} A & -I \\ O & -E \end{pmatrix} = (\mathbf{y},-\mathbf{q})$$

Minkowsky-Farkas theorem implies that the existence of non-negative solutions (**s**, **t**) for the above system of equations is equivalent to say that any **w**, **p** vectors that satisfy the following inequality

$$\begin{pmatrix} A & -I \\ O & -E \end{pmatrix}\begin{bmatrix} \mathbf{p} \\ \mathbf{w} \end{bmatrix} \leq \mathbf{0} \tag{4}$$

satisfies the inequality

$$\left\langle (\mathbf{y},-\mathbf{q}), \begin{bmatrix} \mathbf{p} \\ \mathbf{w} \end{bmatrix} \right\rangle = \langle \mathbf{y},\mathbf{p}\rangle - \langle \mathbf{q},\mathbf{w}\rangle \leq 0$$

⋈

A maximal point of a production possibility set is a point of the set that has no other points in the production possibility set which is greater by the partial order ≤. A maximal point is sometimes called efficient production point. In this paper we use expression “maximal point”. The set of maximal points Ω is called the maximal frontier of production possibility set or simply the production frontier.

Theorem 5.2 (Maximal point of the production possibility set)
Let **y** *be a maximal point of the production possibility set* $P(\mathbf{q})$*. Then, there exist a positive M-column vector* **w** *and a positive N-column vector* **p** *such that*

$$A\mathbf{p} \leq I\mathbf{w} \quad \textit{and} \quad \langle \mathbf{y},\mathbf{p}\rangle = \langle \mathbf{q},\mathbf{w}\rangle\,. \tag{5}$$

Conversely, if a couple of positive vectors **w**, **p** *satisfy the above conditions, then a* *vector* $\mathbf{y}$ *of the* $P(\mathbf{q})$ *is a maximal point of* $P(\mathbf{q})$[15].

Before giving the proof, it is convenient to know a necessary condition for a point be the maximal point.

Proposition 5.3 (A necessary condition for the maximal point)

If a point $\mathbf{y}$ *of the production possibility set* $P(\mathbf{q})$ *is maximal, an activity vector* $\mathbf{s}$*, which satisfies the equation* $\mathbf{y}=\mathbf{s}A$ *also satisfies equation* $\mathbf{q}=\mathbf{s}I$.

(Proof of proposition 5.3)

The proposition can be proved by contradiction. Suppose for a country m, $(\mathbf{s}I)_m<q_m$. As country m's system of techniques γ is productive, there is a non-negative activity level vector such that $\mathbf{t}A(\gamma)=\mathbf{z}$, where $\mathbf{z}$ is a positive vector of R^N and $\mathbf{t}I\leq q_m-(\mathbf{s}I)_m$. Then, if we set $\mathbf{x}=(\mathbf{s}+\mathbf{t})A$, then $(\mathbf{s}+\mathbf{t})I\leq\mathbf{q}$. Then, $\mathbf{x}$ is a point of P($\mathbf{q}$), which is strictly superior to $\mathbf{y}$ with respect to order $\leq$. This is a contradiction. ⋈

(Proof of theorem 5.2)

Let $\mathbf{y}$ be a maximal point of the set $P(\mathbf{q})$. By the definition of maximal point, the inequality

$$(\mathbf{s},r)\begin{bmatrix} A & -I \\ -\mathbf{y} & \mathbf{q} \end{bmatrix}\geq 0 \text{ and } \neq 0$$

has no non-negative solution ($\mathbf{s}$, r), In order to prove this, suppose first that $r>0$. Without loss of generality, we can suppose that $r=1$. Then,

$$\mathbf{s}A\geq\mathbf{y} \quad \text{and} \quad \mathbf{q}\geq\mathbf{s}I$$

and

$$\mathbf{s}A\neq\mathbf{y} \quad \text{or} \quad \mathbf{s}I\neq\mathbf{q}$$

holds. This means that production possibility point $\mathbf{s}A$ is superior to $\mathbf{y}$. In view of proposition (5.3), this is a contradiction to the fact that $\mathbf{y}$ is a maximal point of $P(\mathbf{q})$. Suppose now $r=0$. Then from the second column $-\mathbf{s}\geq\mathbf{0}$. It follows that $\mathbf{s}=\mathbf{0}$, for $\mathbf{s}$ is non-negative. It is impossible then that $\mathbf{s}A\neq\mathbf{0}$. Therefore, it is impossible for any non-negative $(\mathbf{s},r)$ to satisfy the inequalities.

Then, by Stiemke-Tucker's theorem[16], a positive solution ($\mathbf{p}$, $\mathbf{w}$) to the inequalities exists

[15] An equivalent theorem has been presented by Takamasu, 1992, Chap.1 Theorem 2.

[16] To be precise, we use a variation given by Nikaido (1961, p 157) lemma 1.

$$\begin{bmatrix} A & -I \\ -\mathbf{y} & \mathbf{q} \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ \mathbf{w} \end{bmatrix} \leq \mathbf{0}$$

exists, where $\mathbf{p}$ is an N-column vector and $\mathbf{w}$ is an M-column vector. The last statement is equivalent to the conditions

$$A\mathbf{p} - I\mathbf{w} \leq \mathbf{0} \quad \text{and} \quad \langle \mathbf{q}, \mathbf{w} \rangle \leq \langle \mathbf{y}, \mathbf{p} \rangle.$$

On the other hand, as vector $\mathbf{y}$ is a point of the production possibility set $P(\mathbf{q})$, from theorem 5.1,

$$\langle \mathbf{y}, \mathbf{p} \rangle \leq \langle \mathbf{q}, \mathbf{w} \rangle.$$

Combining both inequalities, we obtain

$$\langle \mathbf{y}, \mathbf{p} \rangle = \langle \mathbf{q}, \mathbf{w} \rangle.$$

Conversely, suppose that positive vectors $\mathbf{p}$ and $\mathbf{w}$ exist that satisfy those conditions (5). Take any vector $\mathbf{z}$ satisfying $\mathbf{y} \leq \mathbf{z}$ and $\mathbf{y} \neq \mathbf{z}$, then

$$\langle \mathbf{q}, \mathbf{w} \rangle = \langle \mathbf{y}, \mathbf{p} \rangle < \langle \mathbf{z}, \mathbf{p} \rangle,$$

since $\mathbf{p}$ is positive. From theorem 5.1, vector $\mathbf{z}$ cannot be an element of $P(\mathbf{q})$. This means that the point $\mathbf{y}$ that satisfies (5) is a maximal point of $P(\mathbf{q})$. ⋈

Another simple proposition is useful to clarify how maximal points are located.

Proposition 5.4 (Maximal point of a non-negative direction)

Suppose that the labor-power vector $\mathbf{q}$ *is positive. For any non-negative N-line vector* $\mathbf{x}$ *there exists a maximal point* $\mathbf{y}$ *of* $P(\mathbf{q})$ *that is proportional to* $\mathbf{x}$.

Another useful proposition is on the uniqueness of $\mathbf{p}$ which appears in theorem 5.2.

Proposition 5.5 (Uniqueness of price vector)

Let $\mathbf{y}$ *be the maximal point of the production possibility set* $P(\mathbf{q})$. *If, for a pairing of vectors* $\mathbf{w}$ *and* $\mathbf{p}$ *with* $\mathbf{w} > 0$, *conditions* (5) *hold, then the vector* $\mathbf{p}$ *is unique for each* $\mathbf{w}$.

Proofs of these propositions are simple exercises and are omitted from this paper.

5.2 Production possibility set as the Minkowski sum

In the analysis of the production possibility set $P(\mathbf{q})$, the concept of the Minkowski sum is useful. First this notion should be defined and how the production frontier is shaped will be explained.

Definition 5.1 (Minkowski sum)

Let $S_1, S_2, \ldots, S_K$ be sets of a vector space V. The Minkowski sum of these sets is the set given by the formula

$$\left\{ \mathbf{x} \in V \,\middle|\, \mathbf{x} = \sum_i \mathbf{x}(i), \quad \mathbf{x}(i) \in S_i \right\}.$$

This set is denoted $S_1 + S_2 + \cdots + S_K$ or $\sum_i S_i$.

It is easy to see that the production possibility set $P(\mathbf{q})$ is the Minkowski sum of production possibility sets $P(i, q_i)$ of each country i. It is also easy to see that the maximal point of $P(\mathbf{q})$ is the sum of the maximal points of $P(i, q_i)$. It is important to note that the sum of the maximal points is not necessarily maximal, even if each one of them is selected from each maximal frontier of $P(i, q_i)$.

As we have assumed that each country has a productive system of techniques, the maximal frontier of $P(i, q_i)$ is rather simple. $P(i, q_i)$ is a closed set that has a non-empty interior. Thus $P(i, q_i)$ is an N-dimensional polytope. The maximal frontier of $P(i, q_i)$ is a single facet of $P(i, q_i)$ that has a positive normal vector $\mathbf{p}(i)$. This is another expression of the minimal price theorem, i.e. theorem 3.1. Vector $\mathbf{p}(i)$ is proportional to the minimal price vector of country i. If there is only one technique for each industry belonging to country i and if $q_i = 1$, then the maximal frontier is a $(N-1)$-simplex spanned by N-vectors $\mathbf{a}^1, \mathbf{a}^2, \ldots, \mathbf{a}^N$, if we denote the techniques of industry j by normalized $\mathbf{a}^j$. When there is a choice of techniques, the shape of the maximal facet may have more than N vertices.

More detailed information can be obtained from theorem 5.2. If a point $\mathbf{y}$ of $P(\mathbf{q})$ is maximal, there exists a positive $\mathbf{w}$ and $\mathbf{p}$ which satisfy these conditions

$$A\mathbf{p} \leq I\mathbf{w} \quad \text{and} \quad \langle \mathbf{y}, \mathbf{p} \rangle = \langle \mathbf{q}, \mathbf{w} \rangle.$$

If s is an activity level vector of $\mathbf{y}, \mathbf{y} = \mathbf{s}A$ and $\mathbf{s}I \leq \mathbf{q}$. Then, if for a technique h an inequality $(A\mathbf{p})_h < (I\mathbf{w})_h$ or $\langle \mathbf{a}^h, \mathbf{p} \rangle < w^{C(h)}$ holds, s_h cannot be positive, for

$$s_h (I\mathbf{w} - A\mathbf{p})_h \leq \langle \mathbf{s}, I\mathbf{w} - A\mathbf{p} \rangle = \langle \mathbf{s}I, \mathbf{w} \rangle - \langle \mathbf{s}A, \mathbf{p} \rangle \leq \langle \mathbf{q}, \mathbf{w} \rangle - \langle \mathbf{y}, \mathbf{p} \rangle = 0\,.$$

This means that for a point $\mathbf{y}$ to be maximal there is a pairing of positive vectors, $\mathbf{w}$ and p and $\mathbf{y}$ is a convex combination of techniques that are competitive with regard to $\mathbf{w}$ and $\mathbf{p}$.

Then let Π be the set of all possible shared patterns of specialization. Each element CM of Π has a positive wage-rate vector $\mathbf{w}$ and associated price vector $\mathbf{p}$ such that the set of competitive techniques with regard to $\mathbf{w}$, $\mathbf{p}$ is CM. A production set $P(CM, \mathbf{q})$ with a competitive mode CM is defined by

$$\left\{ \mathbf{x} \;\middle|\; \mathbf{x} = \mathbf{s}A, \mathbf{s} \geq \mathbf{0}, s_h = 0 \text{ for } h \notin CM \text{ and } \sum_{h \in CM \text{ and } C(h)=i} s_h = q_i \right\}.$$

The production set $P(CM, \mathbf{q})$ is a face of the production possibility set $P(\mathbf{q})$. In fact, if vectors $\mathbf{w}$, $\mathbf{p}$ induce competitive mode CM, any point $\mathbf{y}$ of $P(CM, \mathbf{q})$ satisfies $\langle \mathbf{y}, \mathbf{p} \rangle = \langle \mathbf{q}, \mathbf{w} \rangle$ and any element $\mathbf{x}$ of $P(\mathbf{q})$ satisfies $\langle \mathbf{x}, \mathbf{p} \rangle - \langle \mathbf{q}, \mathbf{w} \rangle \leq 0$. The face $P(CM, \mathbf{q})$ is called *modal cell with competitive mode CM*. Thus, the maximal frontier Ω of a production possibility set $P(\mathbf{q})$ can be decomposed into a collection of modal cells that have a different pattern of specialization.

Modal decomposition of the production frontier Ω is a true cell decomposition that satisfies the definition given in section 3.3. In fact, each modal cell is a polytope. Any two modal cells, if they have common point, intersect at a polytope that is another modal cell. Lastly, any point of the production frontier Ω belongs to a modal cell.

In a particular case where countries have no choice of techniques, each modal cell is a Minkowski sum of simplexes. For example, in the case of three commodities modal cells are either triangles or parallelograms.

5.3 Dual correspondence between two modal decompositions

As shown in section 5.2 the production frontier Ω of a production possibility set can be decomposed into modal cells. Then, we have two cell decompositions: modal decomposition of the wage rate Δ and modal decomposition of the maximal frontier Ω of the production possibility set. A natural question is how they are related. As we will see, astonishingly beautiful relations or correspondences exist between them.

There are three levels of correspondences: point-to-point, point-to-set, and set-to-set. In all these correspondences any two corresponding parts have the same competitive mode of techniques when the sets are restricted to their relative interiors.

Let us start from point-to-point correspondence. For any positive $\mathbf{w}$, the minimal price vector $\mathbf{p} = \mathbf{p}(\mathbf{w})$ satisfies the inequality $A\mathbf{p} \leq I\mathbf{w}$. Then, for any positive wage-rate vector $\mathbf{w}$ a net production $\mathbf{y}$ is said to be *admissible* with regard to $\mathbf{w}$ when $\mathbf{y}$ is generated only by a competitive technique with regard to $\mathbf{w}$, $\mathbf{p}$. For a wage rate $\mathbf{w}$ there may exist many admissible productions. So the correspondence from wage rate to admissible productions is not a function from Δ to Ω but a correspondence between Δ and Ω. If $\mathbf{y}$ is an admissible production with regard to $\mathbf{w}$, there exists by definition $\mathbf{s}$ such that $\mathbf{y} = \mathbf{s}A$,

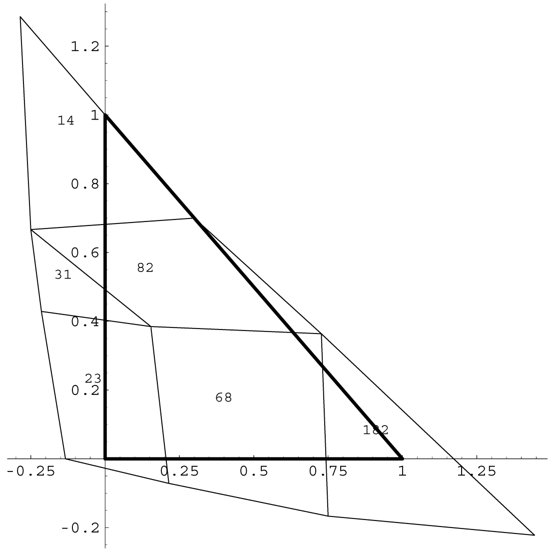

Fig. 4. Modal decomposition of a production possibility set, a projected graph.

$\mathbf{q}=\mathbf{s}I$ and if h is not competitive, then $s_h=0$. Thus the equation $\langle \mathbf{y}, \mathbf{p}\rangle = \langle \mathbf{q}, \mathbf{w}\rangle$ holds. Conversely, if equation $\langle \mathbf{y}, \mathbf{p}(\mathbf{w})\rangle = \langle \mathbf{q}, \mathbf{w}\rangle$ holds for any $\mathbf{y}$ in Ω and $\mathbf{w}$ in Δ, then there exists an activity level vector such that $\mathbf{y}=\mathbf{s}A$, $\mathbf{q}=\mathbf{s}I$ and $s_h=0$ for any non-competitive h. Thus, a necessary and sufficient condition for the admissible correspondence between $\mathbf{y}$ in Ω and $\mathbf{w}$ in Δ is that they satisfy the equation $\langle \mathbf{y}, \mathbf{p}(\mathbf{w})\rangle = \langle \mathbf{q}, \mathbf{w}\rangle$.

Point-to-set correspondence is simpler than point-to-point correspondence. In fact, if we set $Pc(\mathbf{w}) = \{\mathbf{y} \in P(\mathbf{q}) \mid \langle \mathbf{y}, \mathbf{p}\rangle = \langle \mathbf{q}, \mathbf{w}\rangle\}$, $Pc(\mathbf{w})$ is the set of all admissible points of a positive wage-rate vector $\mathbf{w}$. It may well happen that this $Pc(\mathbf{w})$ is empty. For example, if $\mathbf{w}$ does not induce a shared pattern of specialization, some countries do not produce anything and such a production point cannot be maximal. Indeed, it has been assumed that $\mathbf{q}>\mathbf{0}$ and proposition 5.3 asserts that labor inputs of any country i must be equal to q_i. Therefore, it is necessary for a positive wage rate $\mathbf{w}$ to be sharing in order that set $Pc(\mathbf{w})$ be non-empty.

The above correspondence can be shifted up to the set-to-set level. Let $\mathbf{w}(1)$ and $\mathbf{w}(2)$ be two wage rates of the same modal decomposition. They each have corresponding minimal price vectors $\mathbf{p}(1)$ and $\mathbf{p}(2)$, and their sets of associated competitive techniques are identical, i.e. $CT(\mathbf{w}(1), \mathbf{p}(1)) = CT(\mathbf{w}(2), \mathbf{p}(2))$. It is easy to see that $Pc(\mathbf{w}(1)) = Pc(\mathbf{w}(2))$. Thus, one gets a face-to-face correspondence between elements of modal decomposition of wage rate Δ and faces of the maximal frontier Ω of the production possibility set $P(\mathbf{q})$.

Let us now examine the correspondence from the opposite side. Let $\mathbf{y}$ be a point of the production frontier. Then, theorem 5.2 suggests that there are positive vectors $\mathbf{w}$ and $\mathbf{p}$ with the conditions

$$\mathbf{w} > \mathbf{0},\ A\mathbf{p} \leq I\mathbf{w} \quad \text{and} \quad \langle \mathbf{y}, \mathbf{p} \rangle = \langle \mathbf{q}, \mathbf{w} \rangle. \tag{6}$$

Proposition 5.5 assures that $\mathbf{p}$ is unique vis-à-vis $\mathbf{w}$. Thus, $\mathbf{p}$ is the minimal price vector of $\mathbf{w}$. Dual correspondence can be defined as that between the maximal point $\mathbf{y}$ and a wage-rate vector w with $\mathbf{p} = \mathbf{p}(\mathbf{w})$.

Couples $\mathbf{w}$, $\mathbf{p}$ with specific conditions (6) may not be unique for a given $\mathbf{y}$. Let $CT(\mathbf{w}, \mathbf{p})$ be the set of all competitive techniques for a couple $\mathbf{w}$, $\mathbf{p}$. $CT(\mathbf{w}, \mathbf{p})$ varies with different $\mathbf{w}$, $\mathbf{p}$. However, if $\mathbf{w}(1)$, $\mathbf{p}(1)$ and $\mathbf{w}(2)$, $\mathbf{p}(2)$ are two couples that satisfy these conditions (6), there is a third pairing of vectors $\mathbf{w}(0)$ and $\mathbf{p}(0)$ that satisfy these conditions (6) such that

$$CT(\mathbf{w}(0), \mathbf{p}(0)) \subset CT(\mathbf{w}(1), \mathbf{p}(1)) \cap CT(\mathbf{w}(2), \mathbf{p}(2)).$$

Indeed it is sufficient to take

$$(\mathbf{w}(0), \mathbf{p}(0)) = \alpha(\mathbf{w}(1), \mathbf{p}(1)) + (1-\alpha)(\mathbf{w}(2), \mathbf{p}(2)).$$

for a positive α and smaller than 1. This means that there is a minimal set of competitive techniques for any couple $\mathbf{w}$, $\mathbf{p}$ that satisfies these conditions (6). Thus, a unique set of competitive techniques is defined for a maximal point. This will be named the competitive mode of a maximal point.

Definition 5.2 (Competitive mode of a maximal point)
The common set of all competitive technique sets with regard to $\mathbf{w}$, $\mathbf{p}$ *that satisfies conditions (6) is called the competitive mode of* $\mathbf{y}$. *The competitive mode of the maximal point* $\mathbf{y}$ *is denoted by* $CM(\mathbf{y})$.

Once a competitive mode of a maximal point $\mathbf{y}$ is defined, the corresponding set of wage-rate vectors is given by the set

$$\{\mathbf{w} \in \Delta \mid CT(\mathbf{w}, \mathbf{p}(\mathbf{w})) = CM(\mathbf{y})\}.$$

This correspondence can be shifted to face-to-face correspondence between modal decomposition of wage rate Δ and modal decomposition of the production frontier Ω. In this face-to-face level, the correspondence is one-to-one and when the modal decomposition of Δ is restricted to the sharing wage rate vectors the correspondence becomes one-to-one onto map.

Note that a set of competitive techniques $CT(\mathbf{w})$ is uniquely defined for any positive wage rate $\mathbf{w}$. From definition 5.2, a competitive mode $CM(\mathbf{y})$ is given for any maximal point $\mathbf{y}$. If F is a face of the modal decomposition of the production frontier Ω, any point in the relative interior of F has the same competitive mode. This unique competitive mode is defined as the competitive mode of face F.

The same kind of convention is possible for faces of the modal decomposition of wage rate Δ. Any point in the relative interior of a face G has the same set of competitive techniques. We define this unique set as the competitive mode of face G. When the competitive mode of face G is shared, it can also be expressed that G is shared.

By this extension of definitions we can finally make a formal definition of dual correspondence.

Definition 5.3 (Dual correspondence)
Let V be either a point of production frontier Ω, or a face of modal decomposition Ω. Let H be either a point of wage rate Δ or a face of modal decomposition Δ. When V and H have the same competitive mode, one states that V and H are connected by the dual correspondence. It can also be said that V is dual to H and H is dual to V.

To analyze this correspondence in more detail and with rigor, it is inconvenient to see elements of decompositions directly as sets of price space and of production space. It is always necessary to question if the corresponding $\mathbf{w}$ exists or not. A good method to escape from this inconvenience is to take a higher dimensional double space R_{M+N} and R^{M+N}. This is to see a pairing of vectors $\mathbf{q}$ and $\mathbf{y}$ together on the one hand and a pairing of vectors $\mathbf{w}$ and $\mathbf{p}$ together on the other hand.

Let Ξ be the set of all techniques of the world. We set two definitions as follows:

$$Cv(\Xi) = \left\{ (-\mathbf{q}, \mathbf{y}) \;\middle|\; (-\mathbf{q}, \mathbf{y}) = (\mathbf{t}, \mathbf{s}) \begin{pmatrix} -E & O \\ -I & A \end{pmatrix} \text{ for some non-negative } \mathbf{s} \text{ and } \mathbf{t} \right\}.$$

$$Ch(\Xi) = \left\{ (\mathbf{w}, \mathbf{p}) \;\middle|\; \begin{pmatrix} -E & O \\ -I & A \end{pmatrix} \begin{bmatrix} \mathbf{w} \\ \mathbf{p} \end{bmatrix} \le \mathbf{0} \right\}.$$

The set $Cv(\Xi)$ is a $\mathscr{V}$-cone as a non-negative combination of line vectors of

$$\mathbb{A} = \begin{pmatrix} -E & O \\ -I & A \end{pmatrix}.$$

The set $Ch(\Xi)$ is an $\mathscr{H}$-cone as a set of common points of closed half spaces defined by the column vector of $\mathbb{A}$. The main theorem of the polyhedral cone asserts that the two concepts, $\mathscr{V}$-cone and $\mathscr{H}$-cone, are the same. In the sequel, only polyhedral cones are considered.

Two cones $Cv(\Xi)$ and $Ch(\Xi)$ belongs to spaces of the same dimension, but they are included in different spaces that are dual to each other. One is a set of line vectors and the other is a set of column vectors, although we often write them $(\mathbf{w}, \mathbf{p})$ instead of writing them in a column form. Points of two spaces are connected by a bilinear form

$$\langle(-\mathbf{q}, \mathbf{y}), (\mathbf{w}, \mathbf{p})\rangle = \langle \mathbf{y}, \mathbf{p}\rangle - \langle \mathbf{q}, \mathbf{w}\rangle .$$

Writing down matrix multiplication, we see that a point $(-\mathbf{q}, \mathbf{y})$ of $Cv(\Xi)$ has a non-negative activity level vector $\mathbf{s}$ that satisfies the conditions $\mathbf{y} = \mathbf{s}A$ and $\mathbf{s}I \leq \mathbf{q}$. A point $(\mathbf{w}, \mathbf{p})$ of $Ch(\Xi)$ satisfies the inequalities

$$\mathbf{w} \geq \mathbf{0} \quad \text{and} \quad A\mathbf{p} \leq I\mathbf{w} .$$

It is easy to demonstrate the following two propositions, using the Minkowski-Farkas lemma in the case of (2):

(1) If a point $(-\mathbf{q}, \mathbf{y})$ out of origin $\mathbf{0}$ is in the boundary of $Cv(\Xi)$, then there exists a point $(\mathbf{w}(0), \mathbf{p}(0))$ on the boundary of $Ch(\Xi)$ such that a supporting hyperplane Hw is given by

$$\langle \mathbf{y}, \mathbf{p}(0)\rangle - \langle \mathbf{q}, \mathbf{w}(0)\rangle \leq 0 .$$

In addition, as Hw passes $(-\mathbf{q}, \mathbf{y})$, equation $\langle \mathbf{y}, \mathbf{p}(0)\rangle = \langle \mathbf{q}, \mathbf{w}(0)\rangle$ holds. If a vector $(\mathbf{w}(1), \mathbf{p}(1))$ defines another supporting hyperplane and passes $(-\mathbf{q}, \mathbf{y})$, then $(\mathbf{w}(0), \mathbf{p}(0))$ belongs to the same face of $Cv(\Xi)$ that includes $(\mathbf{w}(0), \mathbf{p}(0))$.

(2) If a point $(\mathbf{w}, \mathbf{p})$ out of origin $\mathbf{0}$ is on the boundary of $Ch(\Xi)$, then there exists a point $(-\mathbf{q}(0), \mathbf{y}(0))$ on the boundary of $Cv(\Xi)$ such that a supporting hyperplane Hq is given by

$$\langle \mathbf{y}(0), \mathbf{p}\rangle - \langle \mathbf{q}(0), \mathbf{w}\rangle \leq 0 .$$

In addition, as Hq passes $(\mathbf{w}, \mathbf{p})$, equation $\langle \mathbf{y}(0), \mathbf{p}\rangle = \langle \mathbf{q}(0), \mathbf{w}\rangle$ holds. If a vector $(\mathbf{q}(1), \mathbf{y}(1))$ defines another supporting hyperplane and passes $(\mathbf{w}, \mathbf{p})$, then $(\mathbf{q}(1), \mathbf{y}(1))$ belongs to the same face of $Ch(\Xi)$ which includes $(-\mathbf{q}(0), \mathbf{y}(0))$.

The above correspondence is called *polar relations* in Ziegler (1995), but in the present paper we use the more familiar term “dual relations”, for there is no fear of confusion with duality of oriented matroids. Dual correspondence can be shifted up to a set level correspondence between faces of $Cv(\Xi)$ and $Ch(\Xi)$. A face of a cone is a common set of the cone and a supporting half space. By definition, any face includes the origin $\mathbf{0}$ as its element. A face of a cone is itself a cone. The dimension of a cone C is defined as the maximal number of independent vectors included in cone C and is denoted by $\dim(C)$. The relative interior of a cone is the set of vectors that is expressed as the positive combination of the maximal number of linearly independent vectors of the cone. Note that the relative interior of a half line as a cone is the half line itself.

From this definition, we obtain the next proposition. A faces F_v of $Cv(\Xi)$ and F_h of $Ch(\Xi)$ correspond to each other if and only if there are a relative interior point $(-\mathbf{q}, \mathbf{y})$ of F_v and a relative interior point $(\mathbf{w}, \mathbf{p})$ of F_h such that $\langle \mathbf{y}, \mathbf{p} \rangle = \langle \mathbf{q}, \mathbf{w} \rangle$. When Fv and Fh are in this situation, we also say that face Fv is polar to Fh and vice versa.

Let Fv be polar to Fh, then any vectors selected from Fv are perpendicular to vectors selected from Fh. Conversely, if Fh is polar to Fv, then any vectors selected from Fh are perpendicular to vectors selected from Fv. In this sense Fv is normal to face Fh and conversely Fh is normal to Fv. Moreover, Fv and Fh generate full space in the sense that there is no point $\mathbf{x}$ out of Fv such that $\langle \mathbf{x}, \mathbf{u} \rangle = 0$ for all points $\mathbf{u}$ of Fh, and vice versa. From this, an important formula is obtained for faces that are connected by a dual correspondence.

Theorem 5.6 (Dual correspondence between cone faces)

For any face Fv of $Cv(\Xi)$ there exists a unique face Fh of $Ch(\Xi)$ such that Fv and Fh are connected by the dual correspondence. Conversely, for any face Fh of $Ch(\Xi)$ there exists a unique face Fv of $Cv(\Xi)$ such that Fh and Fv are connected by the dual correspondence. If Fv and Fh are connected by dual correspondence, then

$$dim(Fv) + dim(Fh) = M + N\,.$$

Moreover, if a point $(-\mathbf{q}, \mathbf{y})$ is in Fv and a point $(\mathbf{w}, \mathbf{p})$ is in Fh, and Fv and Fh are connected by the dual correspondence, then

$$\langle \mathbf{y}, \mathbf{p} \rangle - \langle \mathbf{q}, \mathbf{w} \rangle = 0\,.$$

From this theorem we can easily deduce the next theorem 5.7.

Theorem 5.7 (Dual correspondence between modal decompositions)

For any shared face F of modal decomposition of wage rate Δ there exists a unique face G of production frontier Ω such that F and G are connected by the dual correspondence. For any face G of production frontier Ω, there exists a unique face F of modal decomposition of Δ such that G and F are connected by the dual correspondence. If F and G are connected by dual correspondence and N is the number of commodities, then

$$dim(F) + dim(G) = N - 1\,.$$

Moreover, if $\mathbf{w}$ *is an element of F and* $\mathbf{y}$ *is an element of G and if F and G are connected by the dual correspondence, then*

$$\langle \mathbf{y}, \mathbf{p} \rangle = \langle \mathbf{q}, \mathbf{w} \rangle .$$

(Proof of theorem 5.7)

When $Cv(\Xi)$ is expressed as a set of vectors $(-\mathbf{r}, \mathbf{x})$, the production possibility set $P(\mathbf{q})$ is the restriction of $Cv(\Xi)$ to the subspace $\mathbf{r} = \mathbf{q}$. A point $\mathbf{y}$ of $P(\mathbf{q})$ and therefore a point of production frontier Ω is interpreted as a point $(-\mathbf{q}, \mathbf{y})$ of $Cv(\Xi)$. Suppose a point $(-\mathbf{q}, \mathbf{y})$ of Ω is given.

Take a point $\mathbf{y}$ in Ω and a face F that includes $\mathbf{y}$ as its relative interior. As a point of $Cv(\Xi)$, $(-\mathbf{q}, \mathbf{y})$ is included in the relative interior of a face Fv. Then, from theorem 5.6, there exists a face Fh of $Ch(\Xi)$ which is dual to Fv. Take an element $(\mathbf{w}, \mathbf{p})$ of Fh. Since $(\mathbf{w}, \mathbf{p})$ belongs to $Ch(\Xi)$, vectors $\mathbf{w}$ and $\mathbf{p}$ satisfy the conditions $\mathbf{w} > \mathbf{0}$, $A\mathbf{p} \leq I\mathbf{w}$. From theorem 5.6, equation $\langle \mathbf{y}, \mathbf{p} \rangle = \langle \mathbf{q}, \mathbf{w} \rangle$ holds. From proposition 5.5, $\mathbf{p}$ is the minimal price vector associated with $\mathbf{w}$.

Let us examine the dimensions of F and G. First, if a pair $\mathbf{w}$ and $\mathbf{p}$ lies in a face Fh, they can be expressed for an appropriately chosen system of techniques γ by the equation

$$\mathbf{p} = A(\gamma)^{-1} I\mathbf{w}\,.$$

This γ can be fixed as far as the pair $\mathbf{w}$ and $\mathbf{p}$ remains in the relative interior of the face Fh. The dimension of Fh is the maximal number of linearly independent vectors in Fh. Any point of Fh can be expressed as $(\mathbf{w}, \mathbf{p}(\mathbf{w}))$ where $\mathbf{p}(w) = A(\gamma(\mathbf{w}))^{-1} I\mathbf{w}$. Then a set of vectors of Fh has a form

$$\{(\mathbf{w}(1), \mathbf{p}(\mathbf{w}(1)), (\mathbf{w}(2), \mathbf{p}(\mathbf{w}(2)), \ldots, (\mathbf{w}(K), \mathbf{p}(\mathbf{w}(K))\}\,.$$

They are linearly independent if and only if $\mathbf{w}(1), \mathbf{w}(2), \ldots, \mathbf{w}(K)$ are linearly independent. Thus $\dim(Fh) = \dim(G) + 1$, taking in account that w_i are normalized in G by the condition $w^1 + w^2 + \cdots + w^M = 1$.

Elements of face F have the form of $(-\mathbf{r}, \mathbf{x})$. The cone face Fv is a minimal cone in R_{M+N} that includes F. As $(-\mathbf{q}, \mathbf{y})$ is a relative interior of F in a production space R_N, one can take K linearly-independent vectors $\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_K$ when $\dim(F) = K$. One can also take M linearly-independent vectors $\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_M$ in R_M. Then, we can take all points $\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_K, \mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_M$ with origin at $(-\mathbf{q}, \mathbf{y})$ in Fv. Thus, $\dim(Fv) = \dim(F) + M$.

When all these results are combined, we have

$$M + N = \dim(Fv) + \dim(Fh) = \dim(F) + M + \dim(G) + 1\,.$$

As a consequence,

$$\dim(F) + \dim(G) = N - 1\,.$$

When we start from a sharing $\mathbf{w}$ we obtain similar results. Thus, the theorem is proved. $\bowtie$

The price frontier is the set of price vectors that are an image of function $\mathbf{p}(\ )$, which maps a wage-rate vector $\mathbf{w}$ to the minimal price vector associated with $\mathbf{w}$. Function $\mathbf{p}(\ )$ is linear on each face of the modal decomposition of Δ. Two couples of $\mathbf{w}$ and $\mathbf{p}$ belonging to the relative interior of different faces induce different sets of competitive techniques. Thus the dual correspondence can be defined over modal decompositions of price frontiers. Based on what has been observed in the process of the above proof, we obtain the next corollary.

Corollary 5.8 (Price frontier decomposition)

A face H of the price frontier has the same dimension as the corresponding face G of the modal decomposition of wage rate Δ. Let H be a face of the price frontier and F a face of the production frontier Ω. Suppose F and H are connected by the dual correspondence, then

$$\dim(F) + \dim(H) = N - 1.$$

Let $\mathbf{x}$ *and* $\mathbf{y}$ *be two different points of F, and* $\mathbf{p}$ *and* $\mathbf{v}$ *two different points of H, then*

$$\langle \mathbf{x} - \mathbf{y}, \mathbf{p} \rangle = 0 \quad \text{and} \quad \langle \mathbf{y}, \mathbf{p} - \mathbf{v} \rangle = 0,$$

which implies that faces F and H are perpendicular or orthogonal in N-dimensional *space*[17].

5.4 Some simple consequences of the dual correspondence theorem

Dual correspondence theorem is powerful and informative. Let us examine some simple cases.

Example 5.1 (3-country, 3-commodity case)

This is the only case, among many-country, many-commodity cases, where two modal decompositions can both be shown in plane graphics. Here is the correspondence table of faces.

Modal decompositions

production frontier		wage rate and prices
vertex	⟷	*facet*
line segment	⟷	*line segment*
facet	⟷	*vertex*

Example 5.2 (1-country, many-commodity case)

If there is only a one-country, wage rate Δ is reduced to a point. Then, there is only one non-empty face that has dimension 0. If the dimension of the commodity space is N, then dual correspondence theorem 5.7 implies that the production frontier is composed of only one facet that has dimension $N-1$. The price vector dual to this facet is unique up to scalar multiplication. This is the minimal price theorem.

We started from the minimal price theorem (Theorem 3.1). Example 2 shows that theorem 5.7 is an extension of this minimal price theorem.

Example 5.3 (4-country, 3-commodity case)

The wages and prices of this case were analyzed by Graham and others using numerical examples. In this case price space has dimension 3. The wage-rate vector moves in a three-dimensioned simplex. Then, from theorem 5.7 we know that the sharing set of Δ forms a set of 2 dimensions.

Example 5.4 (*M*-country, *N*-commodity case with $M \ll N$)

This theorem is applicable even for large M and N. If a face F of a production frontier has dimension d the corresponding faces of the wage space and price space must have dimension $N-d-1$. Of course this dimension must be less than or equal to $M-1$. Then,

[17] In fact, F and H lie in different dual different spaces of the same dimension and are situated perpendicular to each other.

$d \geq N-M$. When N is much bigger than M, the faces of the production frontier must have dimensions no less than $N-M$. This is clearly a singular cell decomposition. On the other hand, the set of sharing prices has dimensions no greater than $M-1$ in an N-dimensional space. Thus, the prices lie in a very "thin" set. The maximal degrees of freedom are far smaller than the number of commodities. The possibility of price adjustment is limited.

6. Price Adjustment vs. Quantity Adjustment

In the following two sections two major implications of this new construction are examined.

Since Ricardo first published his work until the present, various interpretations of Ricardian trade theory have been presented. J. S. Mill argued for the influences of demand ("reciprocal national demand" theory). F. D. Graham in the second quarter of the 20th century pointed out cases where the changes of demand compositions do not necessarily induce price change and emphasized the importance of the supply side conditions. This controversy continues to the present. For example, Ikema (1993) and Minabe (1995) exchanged opinions concerning the stability of international prices. A long history of analysis can be written concerning this topic. However, my understanding is that the controversy on which of the demand and supply conditions is dominant is misplaced. The true conflict lies in the role of price adjustment and quantity adjustment in international trade situations.

Let us limit our observations on the maximal points of the production possibility set and the wage rates and prices polar to them. My starting point is that, on the production frontier, almost all points lie in the interior of one of the facets[18] (of $N-1$ dimensions) of the modal decomposition. The expression "almost all" is used in a very mathematical sense. Note that the production frontier is composed of polytopes of dimensions less than or equal to $N-1$. Introduce any Lebesgue measure of the ordinary type in which any hypercube of dimension $N-1$ has a positive measure, and then any collections of faces of the modal decomposition are measurable. The set of all faces of dimensions less than $N-1$ has the measure 0. Therefore, there is a significant chance to choose a point in this set that is 0 if we measure probability after introducing the Lebesgue measure.

This leads onto a further discussion. Assume a situation where the net production $\mathbf{y}$ is equal to the demand of the economy. Almost all cases (of probability 1) $\mathbf{y}$ is in the

[18] A facet of a polytope is a face of the highest dimension among faces of the boundary. All other faces of lower dimensions are simply called faces.

interior of a facet. As the interior set is open, any small change in demand keeps the net production in the same facet. The prices and wage rates that are polar to **y** remain constant except for scalar multiplications. This means that in almost all cases price adjustment has nothing to do with the change in demand. In many analyses various explanations have been added concerning production adjustments through price fluctuations. However, when prices return to the original state the only effect of those fluctuations was to produce a change in the activity levels of production to meet the changes in demand. Similar effects can be obtained if prices and wage rates remain constant and quantities are adjusted based on demand. This process is usually called quantity adjustment.

Traditional discussions focused on which the demand and supply conditions was dominant in price determination and in determinations of terms of trade and of patterns of trade. The Ricardian theory of trade suggests, however, that there is a predominance of quantity adjustment over price adjustment.

It appears that this sort of understanding has been accepted since the 1950's. However, the lack of a general theory restrained the scope of analysts and led to much confusion and misunderstanding[19]. People worked mainly either in the price plane, or in the production plane. They could not truly understand how two entities are connected. For example, Minabe (1995, Chap. 8) suggested an overview on Graham's results after Koopmans (1951) publication on the geometry of polyhedral cones. He came very close to the duality theorem. He examined mainly two cases: one case where the state is in (the interior of) a facet and the other where the state is on a 0-dimensional face (i.e. a vertex or a cross-point of ridges). The former was the case observed by Graham: Prices remain constant and quantities are adjusted. The latter was suggested by Minabe and was the case defended by J.S. Mill. In this case, prices may change in a polytope of $N-1$ dimension but the net production or demand must remain at a fixed point. Minabe alleged that this is the case where the " 'reciprocal national demand' theory is completely applicable." However, this symmetric argument of two cases cannot be defended.

There is some misunderstanding concerning the significance of the (strongly) shared

[19] Another example: Ikema (1993) contributed to the advancement of Ricardian analysis in the 3-country, 3-commodity case, developing a method of graphical analysis. He argued about the necessity to include imperfect specialization cases in the analysis ("shared pattern of specialization" in this paper) and yet he could not grasp that his "Graham point" was in fact a general case despite his claim that it is "very special point".

pattern of specialization. As examined in section 3.4, there is in general a wage-rate vector that leads to a strongly-shared pattern of specialization. If such a vector exists, vectors of an open domain around it have the same property. It is conspicuous in the modal decomposition of the wage rate Δ. In fact one or more such domains occupy the major part of the set of sharing vectors. All other faces have dimensions smaller than $M-1$. In the wage rate Δ, the total measure of those faces is 0 if measured by an ordinary Lebesgue measure. However, this is the story of the wage rate and price side. If we look at the production and demand side, as theorem 5.7 suggests, the corresponding faces on the production frontier have only the smallest dimensions, and in the $N=M$ case they shrink to only one or more points. The question is which is more important: Larger sets in evaluation space or larger sets in real commodity space? Once technology and labor powers are given, the most important factors that influence the state of economy are the demands. Of course, demand and prices are connected. One influences the other. However, formal duality stops here. The demand for a commodity is influenced by many factors. The composition of consumer goods is a result of a long history. When one is obliged to choose a state of an economy, demand comes first and prices follow. Reality has greater gravitas than price. Thus, the most ordinary situation is that demand and production drop in the interior of a facet of the production frontier. As examined above, it is the world where prices play a limited role in any adjustment and the major part of adjustment is performed by the quantity changes.

7. Differentials in National Wage Rates

Gains from trade, patterns of specialization and terms of trade are favorite topics of international trade theory. Questions of national wage rate differentials are another important subject of international trade theory, although little enthusiasm has been reported for this topic.

This lack of interest can be traced to the founder of the theory of comparative advantage. When Ricardo discussed his numerical example, he did not mention wage differentials, which should have existed at that time between Portugal and Great Britain. Trade on the basis of comparative advantage is possible only when there are suitable wage differentials. The Heckscher-Ohlin model explains wage differentials as a result of the different ratios of two production factors. When factor price equalization theorem is valid, wage rates must be equalized between countries. In this way, modern trade theory is no more interested in explaining wage differentials between countries. However, this state of interest is quite surprising, because in the 20th century a very large discrepancy has been observed between developed and developing nations. For example, Japanese workers once enjoyed a wage rate 70 times that of Vietnamese workers.

An exception to this was the theory of unequal exchange. Arighi Emanuel (1969) argued that unequal exchange of goods keeps a poor country poor. This theory was welcomed by dependency theory people and became a part of their theoretical reasoning. Despite their good will, they were doubly wrong. Their value theory was based on ethics rather than on economics. They have misidentified the true reasons of underdevelopment. As shown in section 4, trade improves the real wage rate as long as workers remain employed. It is not trade itself that should be accused. Countries should enhance gains from trade while preventing, for a while, loss from trade. To achieve this, it is necessary to control the exchange rate of their currency and prepare their industry for stiffer competition. Despite these misunderstandings, their interest in wage rate differentiation is justifiable. Wage rate differentiations or the disparity in national wage rates is one of the most important topics of trade theory. The new Ricardian theory of the present paper sheds light on this question.

For simplicity of discussion let the demand of every nation be proportional to a predetermined vector. This is equivalent to assume a constant composition of demand for everybody. Then, a maximal production of the world can be determined. In general it can be assumed that this production **y** is in the relative interior of a facet of the production frontier. There is only one wage-rate vector **w** and price vector **p** that are compatible with this production. Here the compatibility means that **w**, **p** and **y** statisfy the equation $\langle \mathbf{y}, \mathbf{p} \rangle = \langle \mathbf{q}, \mathbf{w} \rangle$. That the wage-rate vector $\mathbf{w} = (w^1, w^2, \ldots, w^M)$ is determined means each nation's wage rate w^i can be determined. Thus, the new theory includes explanations why national wage rate differentials occur.

There are many factors that influence wage rate differentials. J.S. Mill argued that prices are dependent on demands. This we cannot deny. When the demand changes and world production moves from one facet to another, wage-rate vector **w** changes accordingly. Change of work power of each country has a similar effect like change of demands. Production frontier changes and corresponding facet may change even when demand composition remains constant. But most important factor in the determination of each nation's wage rate is the technology.

The technology of a country is represented by the set of techniques of that country. Other factors like natural resources and climate influence the set of techniques but more and more in recent years the technology as knowledge, tacit and explicit, determines the possible coefficients of techniques. The level of technology of a country is roughly estimated by the production possibility set of each country. The country that has the biggest production possibility set for a given amount of labor power has the most advanced technology. The story becomes complicated when the ranking differs with the commodities concerned. However, in all cases, we can say this: when labor productivity of a country is increased uniformly by μ and all other conditions are kept constant, the wage rate of the country will be raised to the level μ-times the past rate. Thus, technology matters in the determination of the wage rate of a country. The best way to improve the wage rate is to raise productivity. The wage represents a substantial part of the national income. As the division between wage role and profits shows stability for a long time, to raise productivity is also a good way to increase national income.

In addition, the Heckscher-Ohlin model, at least in its standard model, assumes that two countries have the same techniques and explains that disparity is due to the difference in the factor endowment ratio. In a capital-rich country workers get a better wage rate whereas in a labor-rich country workers obtain less than their counterparts. Factor price equalization theorem, although one cannot question its logical consistency, is also disputable, for wage rate differentiation continues for a long time after trade opens. This might be another valid reason to return to Ricardian theory in place of modern trade theories.

8. Concluding Remarks

The present paper only shows the possibility of a general theory in the Ricardian traditions. As it has been shown that the newly constructed theory covers most general cases: multi-country, multi-commodity, choice of techniques, and trade of intermediate goods. Starting from a given wage-rate vector one finally analyzes how the wage rates are determined. Wage rate vectors and prices are divided into an exclusive set of different competitive modes. The production frontier was also divided into an exclusive set of different competitive modes. Entities with the same competitive mode are mutually related with clear economic meanings. The new construction, presented in this paper, with the help of dual correspondence theorem could clarify the old dispute over the roles of supply and demand conditions. It may also provide an explanation regarding how and why the wage rate differentials are formed and preserved. From such observations an important policy implication can be deducted: the importance of technology and technology development. However, all these are only the starting point of the new theory. Many problems remain to be solved; differences between small and big countries, the effects of tariffs, the effects of transportation cost decrease, the effects of change (not choice) of techniques, wage rate improvement and so on. These are vast subjects and yet I believe a firm foundation has been laid for these new investigations.

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