An Origin of the Neoclassical Revolution: Mill's "Reversion" and its Consequences

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§1. Introduction

In the history of economics, we can detect two major price theories: one is classical theory of value and the other is neoclassical theory of value. The change from classical to neoclassical was the biggest turn in the history of economic theories and is called neoclassical revolution. Factors which paved the way to the neoclassical revolution are numerous. Many explanations were made from externalist point of view, i.e. explanations from the social, political, and philosophical backgrounds. Here I present an internalist explanation, i.e. one by the logic of scientific or theoretic development. I argue that, when John Stuart Mill tried to solve the international value problem, he was forced to revert from classical principle of cost determination of value to a special form of the law of demand and supply. This marked a crucial turning point. The aim of this paper is to prove this contention.

The paper is composed in the following way. Section 2 explains the nature of neoclassical revolution. Section 3 complements section 2 by making essential difference of two value theories more visible. Section 4 argues the nature of Ricardian revolution and situates the neoclassical revolution in a new perspective. Section 5 spots the point of origin of neoclassical revolution and section 6 explains why Mill's "solution" and his thesis for reversion were misplaced. Section 7 gives a bridge between Mill and three founding fathers of British neoclassical economics. Three following sections give individual accounts of Jevons, Edgeworth and Marshall in this order. I will show that all three fathers were deeply influenced by Mill's misleading solutions. Section 11 shows that there were possibilities of alternative way outs. Section 12 summarizes the paper and gives some hints for further investigations.

§2. The nature of the neoclassical revolution

I have not explained how Mill's "solution" was misdirected. It is the task of section 6. Before proceeding to this task, let us see the consequences of Mill's "solution." My bold conjecture is that this was the very point of turn from classical economics to neoclassical economics, or in other expression conversion from plutology (economics of production) to catallactics (economics of exchange). I owe to J. R. Hicks (1974, 1976) this characterization of the neoclassical revolution. This must be the main feature of the neoclassical revolution. Hicks explains this as follows:

> while the classical economists looked at the economics system primarily from the production angle, the catallactists looked at it primarily from the side of exchange. It was possible, they found, to construct a 'vision' of economic life out of the theory of exchange, as the classics had done out of the social product. It was quite a different vision. (Hicks, 1976, p.212) [1]

Hicks defines plutology and catallactics as the difference of the focus of attention. This is an important difference that contrasts two attitudes but it may induce a misunderstanding that plutology or economics of production does not think about exchange. This is a pure misapprehension.

The main difference between economics of production and that of exchange lies on the understanding how the values of goods are determined. As I will argue in the next section, the core of economics is the theory of value. Classical theory of value, best represented by the Ricardian theory, contends that values are determined by production conditions. Neoclassical theory of value assumes that theory of value is possible without any reference to production conditions. Pedagogically it starts from a pure exchange situation. As an inevitable consequence of this abstraction, psychological factors become dominant. It depends too much on the individual agent's ability to deal with total economic processes. There comes the necessity to assume infinite rationality for the agents.

The neoclassical theory of value may be a universal theory that is valid for any economic era and situation, but not a good theory of capitalist economy where producers play predominant roles. The classical theory of value presupposes a capitalist production, or a market economy whose principal players are industrial firms that produce commodities (goods and services) by employing workers and machines. [2] The neoclassical revolution was not only a change of the focus of attention, but a change of logic and structure in the theory of value.

When we talk about neoclassical revolution, we have to take into account that, at the time of neoclassical revolution, economics was not yet a unified discipline that

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[1] Hicks preferred to use the term "catallacticist" instead of "marginalist." I use the expression "neoclassical revolution" to indicate that the revolution comprises much more wide changes of economic thinking.

[2] For more detailed account of classical theory of value, see Shiozawa (2015).

economists in different countries have the same framework of research. Indeed, economics of French speaking countries or that of German speaking countries has very different history than that of English speaking countries. In this chapter, I only examine an origin of the neoclassical economics in the United Kingdom. In the Continental Europe emergence of neoclassical economics and thus the neoclassical revolution took very different courses and we have to observe them from a totally different angle even if we admit that neoclassical revolution was an instance of simultaneous discoveries.

The emergence of neoclassical economics in the UK is a subject most often examined and argued. However, as I have indicated above, majority of explanations were made from externalist views. I advance in this chapter a conjecture that the neoclassical revolution was led mainly by the internal logic of economics. This is totally an internalist view.

This internalist view is necessary because of the peculiarity of the neoclassical revolution in the UK. If the neoclassical revolution is a turn from economics of production to that of exchange, we have to admit that this reverses the order of historical development. In a very old time where trade was only made between communities, exchange rates may have no close relations to the cost of production. Exchanged things were surpluses of communities' necessity. The peculiarity of the neoclassical revolution is that it came after the industrial revolution. The industrial revolution was the most conspicuous event which took place in the time of classical political economy. Why did neoclassical economics, which came after the industrial revolution, ignored this extraordinary phenomenon? It must be even then apparent that tremendous rise of commercial activities is supported by rapid increase of production volumes. Externalist view does not explain this most conspicuous fact.

This chapter advances a conjecture that the neoclassical revolution was a logical process in the development of economic theories. We contend that neoclassical revolution was prepared by John Stuart Mill's study when he wanted to solve an unsettled problem left by Ricardo. It was the question of theory of value in the international trade situation. As this is a conjecture which came to my mind recently (in four or five years ago), and as the stake is extraordinarily big, I do not claim that this chapter provides sufficient evidence to prove my conjecture. It requires enormous work in the history of economics and I have to ask some others to try to

verify or falsify my conjecture. This section remains a rough description of what may have possibly occurred.

John Stuart Mill did not abandon the very core of classical value theory but he was obliged to make a structural reform on the logic of value theory. There were three moments for it. The most important category of commodities that does not obey the law of the cost of production was labor (Mill, Library III.2.19). The second was the joint cost cases (Mill, Library III.16). International values formed the third category (Mill, Library III.16.14). Mill had to admit, if reluctantly, that the law of demand and supply as an anterior and more fundamental law than the cost of production theory of value. This was the first consequence of Mill's "solution". Ricardo once declared that "[t]he opinion that the price of commodities depends solely on the proportion of supply to demand, or demand to supply, has become almost an axiom in political economy, and has been the source of much error in that science." (Ricardo: Sraffa p.382; Library 30.3) Mill knew this and had to reconcile Ricardo's theory of value and his new "solution". What Mill did was to admit the law of demand and supply as more fundamental and logically anterior to the cost of production theory of value. Mill stopped at this point, for this was the maximal possible concession he could make. But economists after Mill did not. They had searched to make their theory more coherent and unified. Here comes the second consequence of the Mill's "solution".

If the law of demand and supply is more fundamental, it is natural to apply this law uniformly and universally. My suspicion is that English founders of neoclassical economics were influenced by the Mill's "solution" and his conclusion. Jevons, Marshall, and Edgeworth show more or less indicative evidences for this suspicion. As Jevons is thought to be the person to have led the marginalist revolution, examination of Jevons will inevitably be longer than Marshall and Edgeworth.

I do not deny that other factors intervened in the arrival of the neoclassical economics. Spread of optimization techniques (in mathematics), tradition of utilitarianism (in philosophy), deep-rooted tradition of demand and supply thinking (in economics), popularization of the concept of functions, and finally the general decline of Ricardian economics. Mirowsky (1989) argued that "energy" concept was crucial to the arrival of neoclassical economics.

All these factors must have worked indeed. But we should ask how direct these factors worked in the formation of exchange economics in place of production economics. Their influences are indirect, whereas impact of Mill's "solution" was, I believe, more direct than others. Above all, Mill was logically forced to accept that the law of demand and supply is anterior and more fundamental than the cost of production theory of value. If we admit that the cost of production theory of value was the center core of the classical economics, it was this internal logic that drove economics of production to that of exchange. All other factors are external to the logic of economics. I agree that the change of society or sciences drive economics to a new direction. However, if economics is an independent science, we should search internal logic that produced the neoclassical revolution. As it is called revolution, it was really a fundamental change of logical structure of economics. It is strange if there is no internalist explanation. My conjecture is one of those scarce internalist explanations.[3]

As far as I know, no one has ever claimed that Mill's "solution," or his situation setting was one of key factors that forced the arrival of neoclassical economics. I believe this conjecture deserves a scrutiny.

§3. Essential difference between classical and neoclassical economics

The core of an economic theory is the theory of value. The theory of value exposes in a most abstract form how an economy works. It exhibits the vision of a specific theory. There are many different aspects between classical and neoclassical economics but the essential difference appears in theories of value. Consequently, in this section, I discuss the difference of two value theories and contrast them.

When we compare two theories of value, classical and neoclassical, the first thing to do is to define what the classical theory of value is, because there is no unified understanding on what it is. As for neoclassical theory of value, there would be no serious confusions. Neoclassical theory of value comprises various strands of

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[3] Hicks (1976) asked himself how we can explain the rise of catallactics. He examined some possible explanations such as reaction to socialism and the change in the real world, but he denied these as major factors and advanced a thesis that the main appeal of catallactics lies in its intellectual quality. Walras made it possible to thinks of economic system as a system of interactive markets and Menger as an adjustment of means to ends. Hicks estimates that Jevons did not completed his system. In this sense, Hicks tries to understand the neoclassical revolution on an internalist standpoint but he does not mention the point I argue in this chapter.

economic thoughts from Marshall and Walras to Arrow and Debreu. However, they have a common core. They are all theories of prices. They put paramount importance on the function of prices. Prices permit the whole complex economy to work, effectively and efficiently. The core of these theories is composed of two parts: one is the utility theory of value and the other is demand and supply equilibrium. No specific structure is required as the theory holds for all situations.

Classical theory of value has no such unified theoretical core as the neoclassical economics has. In the era of classical economics, laws of demand and supply existed. Utility was an important component for many economists. Therefore, main components of neoclassical economics existed already in the time of classical political economy. I do not try to unify various strands in a single framework. If I do it, it would be a total confusion. Instead, I pick up the most typical theory and define it the classical theory of value. I have chosen Ricardo as such a representative. Many people will agree with me. Indeed, Ricardo's theory of value is a culmination of various classical theories. However, there are still many misunderstandings on what Ricardo's theory of value is.

As Ricardo's theory of value, many economists may think of the labor theory of value. As Marx adopted it at the core of his theory, many Marxists would think it granted but my idea is different. My definition of Ricardo's theory of value, and consequently of classical theory of value is the cost of production theory. This claim has textual evidence. Ricardo himself added in the third edition of his Principles a note explicitly claiming this. In fact, he wrote:

> Mr. Malthus appears to think that it is a part of my doctrine, that the cost and value of a thing should be the same---it is, if he means by cost, "cost of production" including profits. (Ricardo: Sraffa, p.47; Library, I., n.7)

The idea is similar to what Marx later argued in terms of production price. We have no space to argue this interpretation here. Read Takenaga (2016) for a more detailed textual examination. As for the possibility of developing Ricardo's idea as a modern theory, see Shiozawa (2016).

Essential feature of cost of production theory of value is its objective character compared with the subjective character of the neoclassical theory. By the word

objective we mean that the value of a commodity is determined by social conditions. The core contention of cost of production theory of value is that prices are primarily determined by the cost of production including profit. In a more modern terminology, this is to say that prices are determined by the full cost principle. If we use mathematical expression, the price vector **p** is given by a system of simultaneous equations:

$$(I + M)\{ w\,a_0 + A\,p\} = p. \tag{3-1}$$

We skip the notations of symbols used here, because they are not used in the following. Those who are interested in this expression are required to consult my paper (Shiozawa, 2016, equation 6).

Many factors are involved in the determination of cost of production. First, production techniques. As a part of knowledge, production technique has a subjective aspect, but a production technique can be specified by input-output relations which are measured objectively. When all inputs that are necessary for the production of a unit of product are known, the cost can be calculated objectively.[4] The expenditure for a kind of inputs is the value times the quantity of the input. If we know the value of all inputs including wages, we can calculate the total expenditure. It is noteworthy that the expenditure is totally observable quantity which has little relevance with toils and pains even if human work is involved[5]. The cost is the sum of all expenditures that were necessary to the production multiplied by a markup factor. The markup factor is 1 plus the markup rate. The total expenditure is to sum up all expenditures and the full cost of the product is the total expenditure times the markup factor, or 1 plus the markup rate. The markup rate is fixed by custom or by calculation for the moment of consideration. We can argue how markup rate changes by the change of market conditions, for example by the change of competitive conditions.[6]

However, to determine the cost of production for all products in an economy is not a

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[4] For simplicity, we assume that production is linear, i.e. inputs and outputs are directly proportional.

[5] There is no necessity to ask if it is proportional to toil and pain (or real cost after Hollander). See Chapter ** (Tabuchi 2017 in this volume).

[6] See for a short account Shiozawa (2016) under the heading *How are markup rates determined?*. Matrix *M* in equation (3-1) is the diagonal matrix whose *j*-th diagonal is the markup rate m_j for product j.

simple operation, because costs depend on values. Here we have a logical cycle. To define the cost of a commodity, we should know the value of all inputs. At the time of Ricardo, system of equations was not widely known and we can easily believe how difficult it was for Ricardo to build a cost of production theory of value. This explains partly why Ricardo often talked about cases where the product is made only by labor. He could include indirect labor but he could not give a precise formula as equation (3-1).

What was more difficult was the question of choice of production techniques. Imagine a situation when you have two production techniques which give the same product. In that case, the superiority of a production technique depends on values. When you know all the input prices, it is the simple question of adding up. However, as a theory of value, you have to determine values by costs and the costs of production depend on values. How does a system of production techniques come to be chosen in such a way that all chosen techniques have the least production cost among techniques that produce the same product?

This is a theorem which was found and proved in the mid 20th century. It was Paul Samuelson who discovered this theorem and named it "substitution theorem" at first and later "non-substitution theorem." Samuelson only proved the two-commodity case and Koopmans proved the three-commodity case and K. Arrow the general case (Koopmans, 1951). Confusion of the naming shows Samuelson did not understand the real meaning of the theorem. I prefer to call this theorem minimal value theorem, because this assures the existence of a system of production techniques that gives the minimal value system given the wage rate.[7] In mathematical expression

$$(I + M)\{ w\, a_0 + A\, p\} \geqq p. \tag{3-2}$$

for all production techniques and equality holds for a system of production techniques that covers all kind of products. Readers of this book will easily notice that the fundamental theorem of the new theory of international values (Chapter 1,

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[7] It is widely quoted that minimal value or non-substitution theorem holds only when no fixed capital goods exist. This is a serious misunderstanding because the theorem is valid in the situation that durable capital goods keeps their efficiency within the depreciation period and are discarded afterwards. See Shiozawa (1975).

theorem 3.4 equation iii) is a simple generalization of (3-2).[8] Sraffa (1960) gave no remarks on this theorem, although he explicitly treated the question in Part III Switch in Method of Production.[9]

As I have argued in Shiozawa (2016), classical theory of value has taken a complete form, only in the latter half of the 20th century. Evidently this is in contradiction to John Stuart Mill's contention that the classical theory of value is complete by 1848 (Mill: Library III.1.2). Although Ricardo could not develop a theory of simultaneous equations or know the minimal value theorem, he knew by his deep insight that the value of a commodity stays constant regardless of sudden changes of the demand or the supply of the commodity and this in the presence of plurality of production techniques. He knew that "prices always vary in the market, and in the first instance, through the comparative state of demand and supply" (Ricardo: Sraffa 119 Library 6.28). However, he knew also that the market price comes back to the natural value, i.e. to the cost of production, after the first disturbance settles down.

In Ricardo's theory of value, demand and supply play no role except that they are only disturbing factors. The values are determined by production conditions together with normal profit margins. If demand changes, production changes correspondingly. If the production adjustment proceeds, the market price returns to the value.[10] In this way, supply and demand have little to do with the determination of values. This abstraction of demand and supply relations from the theory of value is remarkable, because this is to recognize that the essential feature of modern economy is the primal independence of values and quantities. Values are determined by production conditions which are mainly technological. Quantities are regulated by the demand that is requested at the value of the product. This was the real content of the Ricardian revolution. In an industrial society, Ricardo observed that it is the conditions of production that determine value relations. In fact, if the demand of a product changes with whatever reason it may be, the volume of

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[8] In Chapter 1, A is the matrix of net production coefficients modified to an equivalent system. In this chapter, A denotes the matrix of input coefficients. Equation (iii) can be rewritten in this chapter's notation exactly in the same form as (3-2) if vector $\mathbf{w}$ is replaced by a scalar w.

[9] Sraffa may have been aware of this theorem in the form of what he dubbed "Borkiewicz's dictum". See Gehrke and Kurz (2006).

[10] This became much more conspicuous because production adjustment speed became faster in many industries. Modern industrial firms since the 20th century have less necessity to appeal to the price adjustment to adapt themselves to the demand changes.

production can change and adjust it to the changed state of the demand. As far as cost of production remains constant, there is no reason that prices changes.

Few economists, in his time and after him, understand this. Ricardo must have a kind of quantity adjustment ideas. In fact, he argued in this way:

> the proportion between supply and demand may, indeed, for a time, affect the market value of a commodity, until it is supplied in greater or less abundance, according as the demand may have increased or diminished; but this effect will be only of temporary duration. (Ricardo: Sraffa pp.383, Library 30.1)

It is clear that Ricardo assumed that the supply of a commodity or the production would soon be adjusted to the demand. If not, there is no reason that the price would come back again to the original price (natural price). In this sense, Ricardo had in his mind a quantity adjustment mechanism if it was quite different from what we imagine by this term. Classical theory of value presupposes that the supply adjusts to the change of the demand. This is typically a situation of the capitalist economy. Few economists in his time understood that at the back of Ricardo's theory of value a special supply attitude is assumed. It was not written explicitly but is a principle that is twined with Ricardo's price theory (Shiozawa 2016). It is the principle of effective demand at the firm level.

It is here necessary to note that even after Ricardo few people including economists did not understood this very core of Ricardian theory of value. I will argue this point in the next section. The vision that prices regulate discrepancies of supply and demand was an old common knowledge. Ricardo objected to this common sense. John Stuart Mill was a good and loyal interpreter of Ricardo but he was the first man who noticed that the cost of production theory of value did not hold in the case of international trade. Probably after a long reflection, he claimed but cautiously that we must "revert to a principle anterior to that of cost of production, and from which this last flows as a consequence,—namely, the principle of demand and supply" (Mill 1844, Library I.19. Similar expression appears in Mill 1848, Library III.16.5). After Mill, three founding fathers of neoclassical economics (i.e. Jevons, Marshal and Edgeworth) all accepted Mill's contention and began to argue in terms of demand and supply.

My main contention in this chapter is that Mill's "solution" was in fact misdirected and the Mill's conclusion that recommended the reversion to the law of demand and supply misguided the economics to the neoclassical revolution.

Mill opened a tradition to examine an interior vertex of the production possibility frontier. It is the point that I named Mill-Jones point. The tradition continued until very recent days as we shall see in section 6. I myself have been a victim of this tradition when I was trying to write a paper which took form in Shiozawa (2007). This paper was intended to be a successor to my paper in 1985 (written in Japanese) and more than 20 years I tried continuously to find a theorem which might give sufficient conditions for the existence of interior vertices in the models of input trade. Shiozawa (2007) was written when I found a theorem (Theorem 3.4), which I cannot say was a very insightful one. Soon after that, I was liberated from the fixed idea and my obsession fell down. I observed the situation and I finally came to know that John Stuart Mill in the mid-19th century and Ronald Jones in the mid 20-th century were completely misguided by an obsession of price adjustment. A naive simple method to rebuild the international value theory is to focus not on the interior vertex but on the points in the interior of facets. When I acknowledged this, I started to write papers which would compose Shiozawa (2014). The new theory of international values was an extension of the cost of production theory of value. It meant for me a revival of classical theory of value.

After 1960's long discussions were made on the nature of neoclassical economics. Many economists revealed that microeconomics comprises vital flaws. Sonnenshein-Mantel-Debreu theorem (inadequacy of representative agents)、 ubiquity of bounded rationality (impossibility of optimization), non-existence of auctioneer (ubiquity of bilateral exchanges), capital reverse and reswitching (non-existence of scalar quantity of capital), inconsistency of the neoclassical theory of firms (irrelevance of marginal productivity theory), and many other miscellaneous flaws (Beinhocker2006, Keen 2011, Shiozawa 2016). All these flaws come from the basic structure of neoclassical economics. It relies on the individual agents' ability to adjust everything instantaneously. Such an adjustment may be possible for a small economy composed of two or three persons and a few kinds of goods. This is the reason why neoclassical economics textbooks prefer to talk about Robinson Crusoe. All these defects are the results of its basic characteristics: economics of exchange. In opposition to economics of production,

it intends to be a universal theory of economic activities. This abstract nature of neoclassical economics induced it to rely too much on the individual human ability, because it could not assume any social structure that supports economic activities of human individuals. Ricardo's cost of production theory of value assumes a concrete economic structure. This it the reason why it is dispensed to assume omnipotent and omniscient human agents as it is generally done in neoclassical microeconomics. The stake of the option between two value theories is extremely great.

The main stream macroeconomics now normally examines if it has a microfoundation. However, this does not assure in any means that it is a sane economics. Some economists pointed that the macroeconomics of these thirty was spectacularly useless if it is not positively harmful. Minor inventions to adjust and modify macroeconomic models cannot save this state of the art, because it is based on fundamentally flawed microeconomics. As far as microeconomics stays intact, microfounded macroeconomics remains fictive entity deprived of any reality. It is now time to change our paradigm.

This paradigm change necessitates a new theory of value. Fortunately, it exists already. It is the classical theory of value. We can start from it. The classical theory of value had once a grave weak point. It lacked the theory of international values. John Stuart Mill tried to attack this problem and he conceded to the law of demand and supply, because he could not build a theory of international values on the extension of Ricardo's theory of domestic values. The question of international trade was too important to neglect as exceptions. As we will see, this paved the way to neoclassical economics. But now, the theoretical situation has changed. We have a theory of international values which is an extension of the classical theory of value. The classical theory of value had to cede the place of orthodoxy of economics once to neoclassical economics but now can take it back.

In the sequel, we will examine how neoclassical economics emerged from classical economics in the United Kingdom. As I have warned it, I will not treat the neoclassical revolution in Continental Europe or any other places. Before embarking on concrete examination, it will be useful to situate the neoclassical revolution in a wider historical context. We can thus link wormlike procession of individual theoretical efforts and a bird-eye view of the history of economic

theories.

§4. The neoclassical revolution in a wider historical context

The main feature of the neoclassical revolution was a change from plutology to catallactics. It was the change from economics of production to that of exchange. However, this revolution is not a simple revelation of a new truth. In view of classical theory of value, neoclassical theory of value is retrogression to an old common sense. Almost one and a half centuries have passed since the arrival of neoclassical economics. During those years, many components were introduced to refine it. First the concept of marginal utility. Then marginal products. preference order, isoquant curve, decreasing returns to scale, law of diminishing returns (due to input substitution), smooth production functions, constant elasticity functions, representative agent, (unbounded) rationality, rational expectations and many others. Do they prove a tremendous development of economics? Or are they an accumulation of rubbishes which are attached to rescue and beautify the vital flaws of neoclassical economics? Neoclassical revolution must be situated in a wider historical context. For this purpose, let us compare the history of economics with that of sciences.

In the history of sciences, many wrong ideas were accepted as truth for a long period. Many of school textbooks only tell how true theories were discovered by great scientists. However, as Thomas Kuhn (1962, Chapter 1 in particular) emphasized it, truer history is more sinuous and full of deviations. Sciences develop toward a wrong direction for a prolonged period. Wrong ideas persist and even develop in cases. Progress of a science is not necessarily a "development by accumulation". Once an intellectual tradition establishes, if it is true or not, it continues in some cases for a long time before it comes out to be understood wrong.

Phlogiston theory in chemistry survived more than a century until Lavoisier establishes oxidation theory of combustion. Aether hypothesis as conveying medium of light continued more than three centuries until Einstein announces his special relativity theory. Sometimes an erroneous hypothesis came to be strengthened by some most proficient scholars. In the case of Aether, Newton played such a role by assuming "aethereal medium" to explain refraction and

diffraction.

In the time of a science revolution, something different occurs. Kuhn called it paradigm change. Many parts of an old theory are overthrown at the same time. The neoclassical revolution in economics is much more peculiar than common science revolutions. My view on the history of economics is quite inverted from the common understanding. There was once an essentially correct theory but it succumbed to a new theory. The latter proliferated more than one and half centuries and now the old theory is resurrecting as a new hope. Even in the history of sciences we have not many similar stories. Only one story I can cite is heliocentric theory. Everybody knows that geocentrism occupied the mainstream of the astronomy for a long time and reverted by Copernicus and others. This Copernican revolution paved the way to Newtonian physics. However, heilocentrism was not a new idea in a strict sense. We knew that a Greek philosopher named Aristarchus of Samos preached heliocentrism in the Alexandrian age before Claudius Ptolemy wrote or edit *Almagest*. Ptolemic system remained unchallenged more than one thousand years until the arrival of modern science age. Unfortunately, Aristarchus's paper was lost in the history and it was only transmitted that an idiosyncratic scholar called Aristarchus advanced a theory that the earth goes around the sun.

If we consider this story, Ricardo is comparable to Aristarchus of Samos. Since the time of John Stuart Mill until today, a geocentrism called neoclassical economics occupied the mainstream of economics and Ricardo was the subject matter for historians of economic theory. We are now expecting a Copernican revolution in economics.

Some may object to my comparison pointing that Ricardo was much more important and influential economist than Aristarcus of Samos as astronomist. Indeed, Ricardo was once the most reverend economist in England. Keynes once wrote in his *General Theory* that "**Ricardo conquered England as completely as the Holy Inquisition conquered Spain.**" (Keynes, 1936, p.32, Chapter 3, Section 3) Of course, this is a Keynesian rhetoric. It does not prove how Ricardo was accepted in the 19th century. Ricardo may have some strong influence on economic policies but policies and theory are different things. I doubt if Ricardo's theory of value was really understood by a wide range of his contemporary economists.

If Ricardo was established as a firm orthodoxy, it is incomprehensible why oppositions to Ricardo's theory flooded just after his death. A second class economist like Robert Torrens and members of Political Economy Club could claim that all the great principles of Ricardo had been abandoned as erroneous by 1831 (Dobb, 1973, Chapter 4). In England, there were a few economists who understood Ricardo's theory of value. We may count James Mill, John Ramsey McCullock and John Stuart Mill among them. Ricardo's contemporary economists such as Malthus, Jean-Baptiste Say never agreed with or did not understand Ricardo's theory of value. Nassau Senior, whom I will mention briefly in Section 11, was fundamentally against the Ricardo's theory of value (Bowley, 2003[1937], pp.17-19), although his theory is difficult to understand as an integral whole. Ricardo's objectivist theory never really entered in France (Faccarello, 2014). German-speaking world case is not very different from France. Major interpreter did not go further than J. B. Say (Gehrke 2014). This fact partly explains why Karl Heinrich Rau preceded Marshallian cross for many years[11]. Marx praised Ricardo as his precursor and superb analyst of capitalist system but never understood (perhaps intentionally) his cost of production theory of value. If Dobb's estimate is correct, Ricardo's theory of value gloried only about 10 years. Even if we count all years after the publication of Ricardo's Principles until the arrival of the marginal revolution, it is about 50 years. Neoclassical years now count about three times as long as Ricardian years. Ricardo was an ephemeral phenomenon.

Aristarchus of Samos was once a famous mathematician and astronomer, but he was almost completely forgotten by the arrival of Claudius Ptolemy and after that he was the object to be cited as a strange doctrine claimed before Ptolemic system. If we observe Ricardo from a long distance, Ricardo is quite similar to Aristarchus. He is famous as an extreme case of classical political economy, but few economists studied his theory of value seriously.

Keynes's parable of Holy Inquisition is indicative. He only mentions the lack of aggregate demand concept in Ricardo. He was not fair to cite Ricardo in this way. Keynes wanted to make clear contrast between Say and Ricardo in one side and Malthus on the other side[12].

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[11] Baug (2001 p.159) reports that "the first appearance of subjective value theory and a demand and supply diagram-with price on the vertical axis as in Marshall-was in the fourth 1841 edition of Rau's Grundsatze der Volkswirtschaftslehre (1826)."

[12] Keynes cites as economists in the side of Malthus, K. Marx, Silvio Gesell and Major

He was right when he said that considerations on effective demand were lacking in economists before him. If Keynes really understood Ricardo's theory of value and if he had asked advices from P. Sraffa, he could have build his theory of effective demand in a completely different scheme. As I am claiming in elsewhere, theory of effective demand can and should be constructed on the basis of classical theory of value (Shiozawa 2016, section 7; 2017).[13] Keynes missed his chance to define his concept of effective demand on a more firm basis if he accepted Ricardo's theory of value. Cambridge tradition did not permit him to do that. In the beginning of the 20th century, Ricardo's idea was completely wiped out even in Cambridge, England.

Ricardo lived in reality in "the underworld" for a long time and remained as a holder of as curious theory as heliocentrism before Copernicus. My aim in this chapter is to show how this happened despite of appraised authority of John Stuart Mill. Young Mill tried to understand Ricardo's problem and give a "solution" for it. His solution was the real bifurcation point between classical and neoclassical economics. In the following sections I will show how Mill's "solution" was misguided and how his "solution" determined the path of economics after him.

Let me add few more words on Ptolemaic system. I remarked that there is a parallelism between Aristarchus of Samos and heliocentrism in one part and Ricardo and his cost of production theory of value. This parable inevitably implies that neoclassical economics is compared to geocentrism. In these comparison, they have many common points. We know that Ptolemaic geocentric system is a finely constructed exact science. After Copernichus's book *On the Revolutions of the Celestial Spheres* was published in 1543, many eminent astronomers remained to support geocentric system. Tycho Brahe (1546–1601) was one of them. He was a Danish noble and could construct precision machinery for the celestial observation. Aided first by a Danish King and then by a Holy Roman Emperor, he could build observatories, in Denmark and later in Prague. At his life time, he was the most precise and comprehensive observer of the celestial world. This reputation attracted Johannes Kepler to work as assistant to Brahe for a short time. At this time there were many astronomers who supported heliocentric system, including

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Douglas who remained in the underworld.

[13] John Stuart Mill argued that "The demand for commodities determines in what particular branch of production the labour and capital shall be employed." (Mill, I.5.24) This expresses the similar idea as Sraffa's principle of effective demand. For more details, see Shiozawa (2017).

Kepler, but Brahe continued to retain geostatic system. With small variation of Ptolemaic system, he could coherently predict the planetary movements within the observational error range. Heliocentric system became more precise than Ptolemaic system only when Kepler discovered his three laws of planetary motion.

Ptolemaic system was a complex system. Planets move on a circular epicycle whose center moves around on a deferent, the circle with its center near to the earth (the offset is called eccentric). They move at a constant angular velocity viewed from an equant point which is also displaced from the center of the deferent. By this complex configuration he could explain motions of planets, including retrograde motion and change of brightness. To improve the preciseness of prediction, Ptolemy and later astronomers added epicycles to epicycles. In this way, Ptolemaic system became more and more complicated each time a new adjustment was added. In a time some seventy cycles and spheres were employed.

Ptolemaic system and its history are quite similar to the actual neoclassical economics. Neoclassical economics can generate various models and can adapt them to any observed data. It is only less precise than the Ptolemaic system. Theoretical components such as marginal utility, marginal products, preference order, isoquant curve, decreasing returns to scale, and etc. are similar to deferent, eccentric, epicycle, equant and others. These components are invented in order to keep neoclassical system coherent without enquiring the deep meaning of these abstract entities. In the course of one and half centuries it accumulated too many irrelevant theoretical components. Now many economists acknowledge that the neoclassical system has become too complicated and apologetic and contains no deep insights. Key concepts of Ptolemaic system such as deferent, eccentric, epicycle lost value just after the Newtonian revolution. Just like that, after a Copernican revolution in economics, those key concepts such as marginal utility, marginal products, preference order, isoquant curve, and etc. will loose their meaning and will be thrown away in the museum of oblivious concepts.

§5. Mill's unintended "solution"
There is no reason to doubt John Stuart Mill's good faith when he wanted to advance Ricardo's theory one step forward. Ricardo explained how a trade is possible with gains from it for both countries when a country is more productive in

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absolute terms than the other country in both industries. In his explanation, exchange rate was simply given and it was different from that of both countries when they produced without foreign trade. Ricardo's system comprised three theories of domestic values: prices of reproducible goods, prices of non-producible goods and theory of rents and price of lands. In chapter 7 On Foreign Trade, he added a fourth element: prices in international trade.

Ricardo knew well that "the rule which regulates the relative value of commodities in one country, does not regulate the relative value of commodities exchanged between two or more countries" (Ricardo, 1952, p. 133; Library VII.10?). However, Ricardo did not or could not produce any theory concerning how the international values are determined.[14]

As we will see later, the famous illustration using four magic numbers is a repetition of the logic that Viner (1937, p.440, Library) named "18th century rule". Viner is referring to Martyn's reasoning, but it is doubtful if it is appropriate to call the reasoning as "18th century rule," because Martyn (1701) (or Martyn 1720) was not a widely distributed pamphlet and remained obscure until the 19th century when it was re-published by McCulloc (1856). It is true that Adam Smith (Smith 1765, Library IV.2.11-12) gave a similar argument as Martyn's but there is no evidence that Smith read Martyn's pamphlet.[15]

Another point that requires an adjustment in the traditional explanation is that Ricardo had discovered the rule to determine specialization pattern by taking ratios of two coefficients and comparing them. In view of the footnote Ricardo (1951, p.136 , Liberty) made, it is possible that Ricardo compared the two ratios. However, it is doubtful if Ricardo thought the task to determine the specialization pattern an essential part of his trade theory. He gave an example in which we can easily determine it but Ricardo himself was thinking a situation where countries are not completely specialized. As he discussed in later part of the chapter, Ricardo was considering the case where many commodities are traded simultaneously. We find no mentions to treat these cases.

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[14] See Tabuchi (2017?) for the details on how Martin was re-discovered and on the interpretation of Ricardo's argument in his chapter on foreign trade. See also Shiozawa (2016b, section 7).

[15] See Maneschi (2002) for the history of acceptance of the pamphlets. The author of pamphlets was not known until Christine McLeod (1983) established author's identity.

Despite of all these ambiguity, Ricardo's example was ingenious. Paul Samuelson rightly named it Ricardo's four magic numbers. John Stuart Mill started to learn Ricardo from his boyhood. He was particularly interested in problems that he believed Ricardo had left behind him. One such problem was to determine the terms of trade. The terms of trade are an expression to indicate the relative prices between export and import goods. In the two-commodity case terms of trade are the same as relative price or value of two goods in the international market.

As Yukizawa (1974), Ruffin (2002) and Maneschi (2002; 2003) made it clear,[16] Ricardo simply supposed international exchange values that are different from those of domestic exchanges. However, he did not give any hint on how these values are determined and how they are related with each other. He simply remarked that "the same rule which regulates the relative value of commodities in one country, does not regulates the relative value of commodities exchanged between two or more countries." (Sraffa 133, Library 7.10).

Mill found that the terms of trade is not determined in Ricardo's *Principles*. His judgment was right. It is reasonable and justifiable that Mill set the problem in search for the logic that determines the relative value of commodities. Mill might have tried to generalize Ricardo's cost of production theory of value into this international exchange situation. On this point, we have no record which tells how he thought on this. Even if he wanted to do it, it would be a difficult work because, in the time of John Stuart Mill, there were no theories of convex polytopes and linear inequalities.[17]

Without any suitable mathematical tools, Mill and his followers were obliged to work on a simple 2-country, 2-commodity economy. If it was possible to imagine many-commodity case, general analysis of such a case would be highly difficult in the middle of the 19th century. In fact, as we shall see later (in section 11), there were two or three economists who wanted to go beyond 2-commodity case. However, they could not elucidate the reason why 2-commodity and 3- or more commodity cases are different and where the difference comes about.

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[16] Fore the detailed history on this connection, see Tabuchi (2016).

[17] True solution required these theories as it was revealed by Shiozawa (2014),

The trouble with 2-country, 2-commodiy case lies in the fact that there is on the production possibility set only one point where both countries obtain gains from trade. It was the situation of complete specialization. Geometrically it is an internal vertex (extreme point) of the production possibility set and unique point in the positive quadrant (or first quadrant). Mill and his followers did not recognize that this is a singular circumstance which appears only for 2-country, 2-commodity case. We can easily explain how Mill was enforced to consider the economic situation that he has no special intention to examine. Figure 1 gives a standard world production possibility frontier for the case of two-county, two-good case.
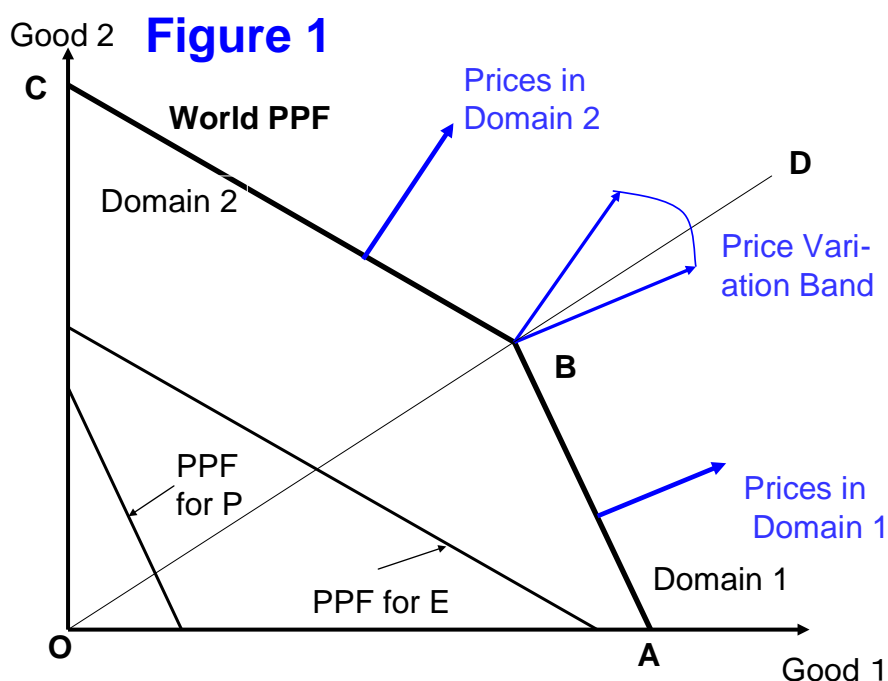

Figure 1

The production possibility frontier (PPF) is the set of maximal points of the production possibility set (PPS). The frontier of two-country, two commodity economy has two segments: segment AB and segment BC. This feature or configuration does not change when the coefficients change. Let us omit two points at the coordinates, because they are degenerated cases. Any point between A and B has a price vector which is identical to the price vector in country P. Any point between B and C has a price vector which is identical to the price vector in country E. In both cases, one country enjoys no gains from trade, because the prices are the

same as those of closed economy for one country.[18] Mill excluded these cases because he thought that trade should be beneficial for both countries. That situation occurs only at the point B. On that point, both countries P and E can enjoy gains from trade. Mill thought that this is the point to examine and asked how terms of trade is determined at that point. Young John Stuart did not know how grave meaning this simple reasoning has for the destiny of economics.

Point B is a point of complete specialization. A country has only one commodity which is competitive in the world market. In this situation, when the labor input coefficients are fixed and each country has a determined quantity of labor (or labor power), the quantity of the product a country can produce competitively is determined. In Mill's example (Mill, 1848, Library III.18.6-17.), Germany specializes in linen and England specializes in broadcloth (III.18.4). If the model is interpreted strictly, then Germany employs all workers to produce linen and England employs all workers to produce broadcloth (Ricardo did not imagined such an extreme case). As the labor power is determined, the quantity of linen that Germany produces and the quantity of broadcloth that England produces are determined. In other words, Mill was lead to observe an economy where two countries engage in the production of one commodity and total amount of the product is determined by the input coefficients and amount of labor force.

This is the situation that Mill found himself in as his international trade example. This is but a pure exchange economy. The above story is of course a logical reconstruction. No explicit words appear in Mill (1848). Nominally, we have productions in Mill's trade economy. However, the amount of production is predetermined and a country can get other commodity only by the exchange of their product. If we pick up the situation where two sides engage in negotiation, it is the same situation where two sides negotiate with each commodity in their back yards.

John Stuart Mill was thus guided by his good faith from Ricardo's production economy to pure exchange economy, which is by their characters totally different

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[18] From the figure itself, it is note clear that wage rates of both countries are determined uniquely at the same time as prices. The international value system comprises wages of all countries and it is uniquely determined when the world demand lies in the interior of a facet, or on an open domain of the frontier. See Theorem 3.4, Chapter ×× of this book.

from what Ricardo examined.

How Mill solved the determination problem of terms of trade is not of the first importance. The most important fact is that Mill displaced the problem from production economics to exchange economics. He unconsciously did this. I contend that this was the very point where economics of production of the classical political economy was replaced by economics of exchange in the neoclassical tradition. After Mill, most of trade theorists continued to examine the situation that Mill had set and could not escape from it except for a few exceptional cases.

§6. How is Mill's "solution" misplaced?

Now it is time to argue Mill's "solution." Was it a really solution to the Ricardo's problem of constructing a theory of international values. Was Mill's "solution" a unique possible solution to the theory of international values? By no means! Mill's "solution" was an observation of an exceptional case. Mill and many other economists in international trade theory mistook this exceptional case as a representative situation of international trade. Let me explain first by a figure.
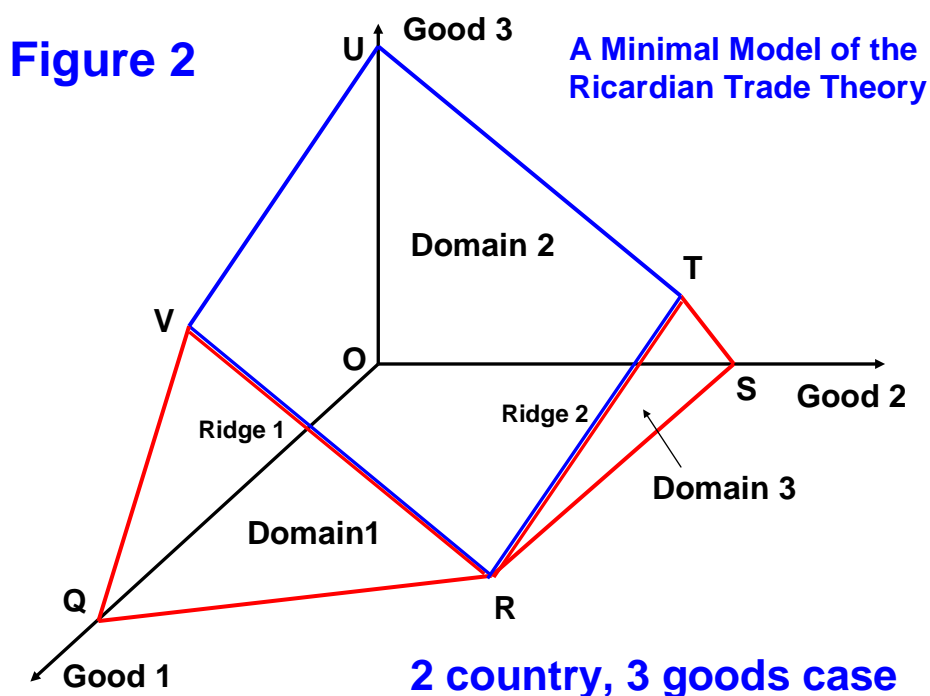

Figure 2

Figure 2 presents the world production possibility frontier (PPF) in the case of

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2-country, 3-commodity case. It is named minimal model of the Ricardian trade theory. I will justify this denomination soon.

The world PPF of Figure 2 consists of three facets (open domains), eight ridges and six vertices. The first observation is that there is no vertex in the interior of the positive orthant. No point like B in Figure 1 exists. We call such internal extreme point Mill-Jones point[19]. Formal definition of Mill-Jones point is any vertex of the world PPS that lies in the interior of the positive orthant. Let us call such a point of PPF interior vertex. Coordinates of such a point must be all positive and is an extreme point of the world PPS (We omit the modifier "world" here after).

As we have observed, there exists no Mill-Jones point in Figure 2. One may think that this is a mere incidence, but it is not. No trade model has Mill-Jones point if the number of commodities exceeds the number of countries. An internal vertex represents a situation of complete specialization. This means that every country has only one commodity which is competitive in the world. Then, there is a one-valued mapping from set of countries to the set of commodities. Therefore, if a Mill-Jones point exists, the number of commodities is equal to or less than the number of countries. In Figure 2 case, the number of commodities is greater than the number of countries and it is impossible to have an internal vertex. In general, if the number of commodities is greater than the number of countries there exists no Mill-Jones point.

There are vertices at the boundary of the PPS. At these points, one or more coordinate vanishes. It means that one or more commodity is not produced at all. These are degenerated cases and we do not consider them, because they are the situation where some commodities are not produced at all. We cannot say that the number of produced commodities is 3 in this situation.

Mill-Jones points or interior vertices do not exist when the number of commodities is greater than the number of countries. This simple proposition went unnoticed nearly 150 years after J.S. Mill. But it has an important consequence. We can count about 200 countries or economies in the world. The number of commodities,

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[19] Some call this point "Ricardo point" or "Ricardo's Limbo point". Because these are not a suitable naming, I do not adopt this naming. The reason of my naming is given later.

although it is difficult to count them exactly, easily exceeds millions. For example, a price book of the Gosplan in the former Soviet Union, it is said, contained more than 20 millions of items. We may assume any developed economy trades and produces more products than that. We may safely assume that the number of commodities is bigger than the number of countries. Non-existence of Mill-Jones points means that there is no pint at which prices can move freely within an open range. People after Mill continued to try to determine prices on the assumption that they have some margin of free movement, but there is no such possibility when the number of commodities is bigger than the number of countries.

What happens in a point other than Mill-Jones point? If we see Figure 2 again, we easily know that most point of the PPF lies in either of three domains. In Domains 1 and 3, the prices are proportional to those of a country. J.S. Mill excluded this situation considering that if trade continues, both countries must have certain gains from trade. In the two-country, two-commodity case, such a point is unique. It is point B of Figure 1. In case of two-country, three-commodity case, we have a wide Domain 2. Any point in Domain 2 has a system of international values which remains constant as far as the point stays in Domain 2. This system of international values is different from either of two countries' domestic values and two countries (and employed workers in both countries) can enjoy gains from trade.

There is a possibility that the world production or demand comes on one of two interior ridges RV and RT. We can exclude the points on the border of the frontier, as they are degenerate points. At a point on RV, for example, the prices can vary but they must remain perpendicular to the ridge RV. The degree of freedom of those prices is only one dimensional. Moreover price changes have no effects to move the demand and the production of the world. The price variation changes the exchange ratio between good 1 and a combined set of goods 2 and 3 but has no effect to change the demand from a point on RV to another point on RV. In fact, Ridge RV is parallel to ridge TU and to the plane that is supported by axes OS and OU. All points on RV have the same quantity of good 1. The same explanation holds for ridge RT. Then, price adjustment does not work at any point on the frontier.

Domains 1 and 3 are the cases which are sometimes studied as big country cases. Although Domain 2 in Figure 2 stands in a similar situation like point B of Figure 1, they present very different characteristics. At point B the prices moves freely within

a certain range (the degree of freedom is the same as the number of commodities minus 1) and the world production is fixed. At any point in Domain 2, the price system remains constant and the world production can change freely in the domain. These characteristics are quite similar to the classical value theory. There is only one value system and supplies can be adjusted to any effective demand as far as it stays in Domain 2.

Now the whole picture becomes clear. Mill examined a two-country, two-commodity case believing that the model gives a representative situation and fell in an unexpected trap.
He did not imagine that the situation changes drastically and in an essential way when the number of commodities is bigger than the number of countries. If we admit that the number of commodities exceeds the number of countries, the minimal model of international trade should be 2-country, 3-commodity case. This is the reason that I call Figure 2 minimal model of the Ricardian trade theory.

John Stuart Mill started a tradition to consider complete specialization case. It meant at the same time to examine a pure exchange economy, because in such a case the product and the volume of each country's production is determined and the countries (or representative agents) negotiates the exchange ratios between commodities. This tradition has been succeeded by economists after Mill and it remained so even at the middle of the 20th century.

In 1950's there was a kind of resurrection of Ricardian trade theory. Following the works of Graham, Lionel McKenzie and Ronald Jones extended the Ricardian model from 2-country, 2-commodity case to many-country, many commodity cases. This was a great step forward in the Ricardian tradition but the tradition was conserved by them. McKenzie wrote three-odd papers on Ricardian trade model and Jones followed him. Their works marked a new era as their analysis was based on the new mathematical tool developed by the activity analysis which was a topical trend at that time. Curiously, McKenzie treated the cases where the number of countries is equal or greater than the number of commodities. Jones (1961) discovered a famous formula that I will explain briefly soon after. The formula gives the necessary and sufficient condition that a pattern of full specialization is realizable ("efficient assignment" in Jones's terminology). This formula proves that there is at most one internal vertex in the case of N-country, N-commodity case. Jones also considers

unequal cases, i.e. the case where the number of countries is not equal to the number of commodities. However, as his definition of "class" (Jones 1961, p.164) shows, he is interested in cases where the number of countries is greater than the number of commodities. Why did McKenzie and Jones examine such singular perverse cases, whereas a common sense or a simple observation tells that number of commodities is far greater than the number of countries?

Jones's formula is expressed in the form of next theorem.

[Jones Theorem](Jones's formula)[20]

Let a_{ij} be labor input coefficients for country i to produce commodity j. Suppose there are N countries and N commodities. Then the following two conditions are equivalent:

(1) There exist positive wage rates w_1, w_2, ... , w_N which satisfy

$$w_1 \cdot a_{11} < \ w_2 \cdot a_{12}, w_3 \cdot a_{13}, \cdot \cdot \cdot \ , w_N \cdot a_{1N};$$

$$w_2 \cdot a_{22} < \ w_1 \cdot a_{21}, w_3 \cdot a_{23}, \cdot \cdot \cdot \ , w_N \cdot a_{2N};$$

$$\cdot \qquad \cdot \qquad \cdot$$

$$w_N \cdot a_{NN} < \ w_1 \cdot a_{N1}, w_3 \cdot a_{N3}, \cdot \cdot \cdot \ , w_{(N-1)} \cdot a_{N(N-1)}.$$

(2) For any permutation τ of N indexes which is different from identity,

$$a_{11} \cdot a_{22} \cdot \ \cdot \cdot \cdot \ \cdot a_{NN} < a_{1\tau(1)} \cdot a_{2\tau(2)} \cdot \ \cdot \cdot \cdot \ \cdot a_{N\tau(N)}.$$

The second condition can be translated that left side product is the strict minimum among permutation products in the form of the right side. The condition (1) is equivalent to the existence of internal vertex or positive extreme point.

This is a beautiful generalization of Mill's formula (comparison of two ratios from four magic numbers) for comparative advantage. In Mill's case, country 1 specializes in commodity 1 and country 2 specializes in commodity 2 when two ratios satisfy the following relation:

$$a_{11} / a_{12} < a_{21} / a_{22}. \qquad\qquad (5\text{-}1)$$

Here a_{ij} are labor input coefficients. The above condition is equivalent to

$$a_{11} / a_{21} < a_{12} / a_{22}. \qquad\qquad (5\text{-}2)$$

These two conditions are in this turn equivalent to

$$a_{11} \cdot a_{22} < a_{12} \cdot a_{21}. \qquad\qquad (5\text{-}3)$$

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[20] Jones proved the proposition (1)⇒(2), but it is doubtful if Jones proved the inverse proposition (2) ⇒ (1). Jones (1961). See Shiozawa (2015) Section 10.

Jones theorem tells that above three equivalent conditions (5-1), (5-2) and (5-3) are also equivalent to the existence of positive numbers $w1$ and $w2$ which satisfies the conditions

$$w1 \cdot a11 < w2 \cdot a21 \quad \text{and} \quad w2 \cdot a22 < w1 \cdot a12. \qquad (5\text{-}4)$$

This equivalence theorem can be demonstrated as follows. If positive numbers $w1$ and $w2$ which satisfy (5-4) exist, multiplying both sides of the two inequalities, and eliminating $w1 \cdot w2$ from both sides, we get (5-3). The converse holds too. In fact, if assume condition (5-2) holds, take positive numbers $w1$ and $w2$ in such a way that

$$a11 / a21 < w2 / w1 < a12 / a22.$$

Then we can derive the first inequality from the left inequality of (5-4) and the second inequality from the right inequality of (5-4).

The left inequality of (5-4) means that the cost of production of commodity 1 in country 1 is cheaper than the cost of production of commodity 1 in country 2. In other words, the production of commodity 1 in country 1 is more competitive than in country 2. In the same way, the right inequality of (5-4) implies that the production of commodity 2 in country two is more competitive than in country 2. With wage rate $w1$ and $w2$ which satisfy (5-4), we get an economy in which country 1 specializes in commodity 1 and country 2 specializes in commodity 2. Thus the above equivalence theorem implies that real cost and money cost approaches are in fact equivalent in the case of 2-country, 2-commodity case.

Jones theorem gives a sufficient condition for the existence of a Mill-Jones point. The theorem also implies that if Mill-Jones points exist at all, they are unique. We can thus observe a keen interest on Mill-Jones point at the time of Ronal Jones. However, as we have observed, no such point exists when the number of commodities N is bigger than the number of countries M. If $M=N$, there exists at most one Mill-Jones point. If $M>N$, there may exist many Mill-Jones points. In fact, there are in a general case as many Mill-Jones points as the number of "classes". If Jones knew all these facts, why did he not consider the non-existence case? It must be easy for him to reason that internal vertex does not exists in the case $M<N$. This strange fact proves how the tradition Mill opened was strong and binding.

Mill started a well established tradition that he may not have intentioned. Mill was guided to this tradition simply by the easiness of examination, but the stake was big.

This simple examination marked the start of new economics later named neoclassical economics. Jones was a deep thinker but was fully immersed in the tradition of neoclassical economics. He was too preoccupied by the price adjustment framework to see the most common cases. They were not suitable to price adjustment paradigm and they were exclude from observation. In the time of classical political economy, this was not such an established tradition. John Stuart Mill, intended or unintended, set the problem which was not solved by Ricardo and thus started new economics of exchange.

§7. Mill's "solution" and after

John Stuart Mill posed his question. How solved it is not important. What matters is how he posed it. We know his solution well. The international values come to an intermediate point of two extremes that are values of each of two countries. He showed how, at a value, two countries would exchange their products against the products of the other. Existence is not discussed in a modern way. In sections 6 to 8 that were added in the third edition, Mill started arguing the question of multiple equilibria, but he did not give any sufficient condition for the a solution to be unique. These questions are studied by Alfred Marshall and we will give a short comment in the section on him.

Despite of this inadequate character of Mill's "solution", I have no intention to reprimand the lack of exactitude of his solution. At the time of John Stuart Mill i.e. at the middle of the 19th century, mathematics itself was still intuitive than logical. Even many-year later, Jevons and Walras were satisfied by counting the numbers of equations and unknowns. What is more important and crucial to the history of economics is what he believed to have established in his *Principles*. It is sometimes difficult to distinguish the influence of his "solution" to the unsettled trade question and other observations he made in the course of his economics formation. Although I admit that the followings may not be exclusive result of his "solution", we can observe that Mill made a tremendous step forward towards the economics of exchange in the course of considering and examining the international value problems. Here is a list of propositions that Mill came to affirm in the course of this examination:

(1) It is not the absolute advantage of an industry but comparative advantage that determines the pattern of trade and specializations. (*Principles* III.18.60; *Essays* I.2,

I.9)

(2) The law of demand and supply is anterior and more fundamental to the law of cost of production (*Principles* III.16.5; E I.19).

(3) The price regulates itself in such a way that demand will be equal to supply (*Principles* III.18.15, a citation from *Essays* I.32; *Essays* I.23).

(4) Supply and demand are but another expression of reciprocal demand. (*Principles* III.18.24)

(5) Demand depends on value (or price). (*Principles* II. ; *Essays* I.38)

(6) Equation or equilibrium of international demand is the equality of exports and imports. (*Principles* III.20.5; *Essays* I.52, I.72)

(7) Each country completely specializes in the production of one commodity. (*Principles* III.18.4; *Essays* I.6)

(8) 2-country, 2-commodity analyses can be generalized without fundamental change of logics. (*Principles* III.18.17, III.18.20, III.18..22, III.18.61, III.18.63; *Essays*)

Mill believes that proposition (1) is what Ricardo wrote in his Foreign Trade chapter. However, as it was made clear, Ricardo did not explain in this way at least on his four magic numbers (See Yukizawa 1974, Maneschi 2003, Foccallero 2015, Tabuchi 2016).[21] This is practically John Mill8s invention. And yet his idea is still influencing the international trade thinking. Proposition (2) became a kind of manifesto for the neoclassical revolution. In *Principles* Mill advances this statement in a chapter 16, Book III, Some Peculiar Cases of Value, but he put it here, in my guess, to attenuate too drastic an announcement for many Ricardian loyalists including Mill himself. However, what Mill prepared in the International Value chapter was not this manifesto alone.

Proposition (3) was the real content of proposition (2). In the *Prinicples*, there is no explanation what is the principle of demand and supply, but in *Essays* Mill explains that the principle of demand and supply means that price or exchangeable value depends on demand and supply (*Essays* I.23-24). In the *Principles*, Mill makes no such hint before International Value chapter except two minor comments i.e. a part that was omitted in the third edition where he spoke of the adjustment of wages

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[21] Ricardo's footnote on shoe maker and hat maker (Sraffa 1951 p.138) is more ambiguous. He may be thinking as we normally do by taking two ratios of two different industries, each for two countries.

(note #84) and he talks about adjustment of prices of two jointly produced products (*Principles* III.16.15). On the contrary, he emphasizes the opposite causal relation at the beginning of the Book III:

> [T]he value of things which can be increased in quantity at pleasure, does not depend (except accidentally, and during the time necessary for production to adjust itself,) upon demand and supply; on the contrary, demand and supply depend upon it. (*Principles* III.3.7)

The notion that value depends on supply and demand already appears in the *Essays* (I.23). The essays were written in 1829-30 (P.1), but examination on how gains from trade are divided between trading countries went back to early 1820's when Mill was still teen agers. Mill knew very well that the principle of cost of production was the core of Ricardo's whole doctrine. He discovered by his study on the international trade questions that this central dogma had a deep crack. Knowing this from his young days, Mill remained at surface a loyal adherent of Ricardo's theory. He proposed that we should revert to a more general and fundamental law of demand and supply. In the *Principles*, this proposal was placed only after the main argument on values was over and just before the important two International Trade chapters. Despite of this careful presentation of his new doctrine, students after him have found that this is the key point that may revolutionize the whole system of economics.

What Mill named peculiar cases may have given some influence on adopting the new doctrine. For example, Alfred Marshall studied the cases of joint cost of production in his early days when he started to study economics as his major research field. However, the importance of international values is without comparison. As we will see in the coming three sections the three pioneers of British neoclassical economics have picked up international trade problem as one of major research targets.

International trade situation as it was set by John Stuart Mill was more helpful than we imagine in the formulation of neoclassical economics. Mill may have not been well aware of the effects of complete specialization, even though he clearly states this point as proposition (7). In a 2-country, 2-commodity case, what each country produces and how much it produces are determined as we have observed in

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section 5. This introduces a pure exchange situation. It is evident that this situation setting paved the way to the economics of exchange.

In addition, this situation helped to establish the symmetry of demand and supply. In this 2-person 2-good pure exchange economy, supply is in fact the demand of another good. As Mill put it, supply and demand are in fact an expression of reciprocal demand. This helped much the formulation of supply curve, because the supply was symmetric counter part of the demand. The notion of demand curve or function was easily formulated by the utility maximization. Supply curve or function, when the production intervenes, was much difficult to formulate. Marginal principle was first introduced for utility, but its introduction in production was retarded many years. We know how Marshall struggled to harmonize supply curve with the production conditions. He invented the notions as internal and external economies. He barely succeeded to explain increasing supply curve by excluding economies of scale within a firm. Such kind of complication did not existed in the case of reciprocal demand.

When we acknowledge that demand depends on prices, it is easy to know that supply depends on prices in the case of reciprocal demand. This acknowledgement established symmetric framework of supply and demand. Although it seems Mill did not arrived at the notion of demand and supply function with prices as independent variables (Yoshii, in this volume), there was only a step to that. What was lacking for Mill was the mathematical concept of a function in general. In the time of John Stuart Mill, an idea of function in general was not very common. Mathematicians may have used such an abstract concept, but for non-specialists a function was something expressed by algebraic expressions.[22] It must be difficult for Mill to conceive the relation between the demand and the price as a function.

Mill's concept of equilibrium of international demand was a bit special one. It meant the trade balance and in this sense equilibrium did not signify the equality of demand and supply as it means in the modern sense. We will see in section 10 (p.48 in the draft) that he must be thinking of a kind of cybernetic process. However, the *Principles* wiped out some of traditional misconceptions. As Mill put it, many

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[22] This explains in part why A. A. Cournot (1838) had introduced the concept of demand function as early as 1838. Cournot was trained as mathematician, In addition, mathematics education was much developed in France than in England in the first half of the 19th century.

persons assumed that value depends on the proportion between the demand and the supply. He pointed that it is not the proportion but the equality of them (III.3.5). However, this equilibrium concept may have induced a grave slip when Mill considered the situation where more than 2 country or 2 commodity cases are examined. He suggests that these cases can be treated just as the same way as the case of 2-country 2commodity case (III.18.17; III.18.61), but it seems he did not reflected on these questions deeply. He claims that the introduction of third country or commodity does not alter the theory. He is right in one sense. In his understanding total value of exports and imports must be equal. He is preoccupied in this equality or trade balance and does not think how the specializations are defined in these situations. It is difficult to suppose he has any concrete idea in mind, because this question remained difficult question even at the middle of the 20th century. Curiously, Mill does not mention explicitly that his two by two case analysis can be generalized to many country or commodity cases in the first of Essays. It is probable that he knew the difficulty when he was working of the specialization and had forgotten some twenty years later when he started to write the *Principles*.

As a simple conclusion of this chapter, we may say that International Value chapter prepared most of the neoclassical frameworks and marked a clear shift from economics of production to that of exchange. In chapter 3 of book 3, he stated clearly that value of things does not depend upon demand and supply, but demand and supply depend upon it. In chapter 18 of the same book, Mill set a situation where the exchange value adjusts itself so that the demand and the supply become equal. In the former, the mechanism that brings the supply equal to the demand was the change of production volume. In the latter, the production was put off and a pure exchange situation was introduced. In this state, two parties have certain quantity of each of two commodities. In the course of long explanations, a clear change of adjustment mechnism occurred and the causal orientation was reversed. Without mentioning this change of adjustment mechanisms, and without investigating why this change of adjustment occurred, Mill declares that the law of demand and supply is more general than that of cost of production. As we have explained in section 6, there was a grave oversight. We will see in the following three sections how this oversight guided late comers to the construction of economics of exchange. More precisely it was the economics where the exchange plays the major role whereas production is interpreted just as a variant of exchange

(production as exchange with the nature).

§8. William Stanley Jevons

In the case of Jevons, my judgment is symptomatological. There is no textual evidence that Jevons was influenced by Mill's "solution". Despite his apparent hostility and contending attitude against John Stuart Mill, and although there are no direct evidences, it is highly probable that Jevons unconsciously accepted Mill's fundamental framework in international trade.

Let me start my argument by citing two interesting papers. One is Donzelli (2007) and the other is Aldrich (2000). Donzelli claims that Jevons did not develop law of demand and supply. Aldrich (2000) argues the reasons of non existence of Jevonian revolution in international trade theory.

 Let us see what Donzelli (2007) questions:

> One of the most surprising features of Chapter 4 of TPE [*Theory of Political Economy*] is that, in spite of the reiterated emphasis laid by Jevons on the allegedly fundamental role played by the so-called "laws of supply and demand" in his theory of exchange, no formal demand-and-supply analysis is actually employed by the author in deriving such theory nor, in spite of what Jevons himself occasionally claims, can be deduced from the formal statement of the theory, as can be found in TPE. (Donzelli 2007, p.2)

It is true that expressions "demand function" and "supply function" do appear in Jevons (1871) (See also White, 1989; Nakano, 2009). In the Preface to the second edition, Jevons refers to what we call "demand function" when he talked about Cournot, but he cite it as "a function of the price, or $D = f(p)$". Even in his mathematical theory (1874) no such expression appears. Jevons was trained as natural scientist, particularly good in chemistry, and had a concept of function in general. He uses the term such as "function of utility" but he never used the concept of "demand function" or "supply function" in his principal book as his own concept. What does this mean?

A simple explanation is that Jevons did not think in terms of demand and supply

functions. If Donzellii (2007) pointed this fact, he is right. Then, what did Jevons mean by the laws of supply and demand? [23] In the Preface to the second edition of his *Theory*, Jevons cites Lardner (1850) that he found in it a "mathematically and graphically" treated example of the laws of supply and demand. The chapter 13 indicated by Jevons is an account of receipts and profits.[24] A diagram appears in p.249. Lardner discusses how the tariff influence the profits and argues that there is a maximum point of profit when the tariffs are continuously increased from a very low value to higher values. These must be a good example of mathematical treatment of profit analysis and it must be true that Jevons got an "idea of investigating Economics [sic] mathematically". However, it is difficult for us to see that this is related to a law of demand and supply. What Lardner argued there is a calculation of profit maximization.

In the body of the book, expression of "laws of supply and demand" appears ten odd times. In any of them, there is no detailed explanation about the laws. Many of them indicate that the laws of supply and demand are the consequences of law or theory of exchange (*Theory*, Library I.2, I.24, IV.39). The most detailed account of the laws appears in Chapter I Introduction. In the second appearance Jevons explains: "The ordinary laws of supply and demand treat entirely of quantities of commodity demanded or supplied, and express the manner in which the quantities vary in connection with the price." (I.5) This explanation of the law of demand and supply does not seem to differ much from the commonly accepted notion of the law. However, we may infer that there is a shift of main focus between what John Stuart Mill meant by law of demand and supply and what Jevons meant by laws of supply and demand. By these expressions, Mill mainly meant the equality of demand and supply (See Yoshii, this volume). In the case of Jevons, he may have implied the equality of supply and demand, but it seems he was more concerned in the form of how demand and supply changes with the change of the price. There also seems be a discrepancy between what Jevons understand by demand change and what we imagine by the same expression. When we think of demand function (or curve), we normally think a function between the demand and the price. This kind of conception is very rare in Jevons. Instead, Jevons talks about utility function (or

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[23] Expressions law / laws of demand and supply do not appear in Jevons (1871). By contrast, the expression "law of supply and demand" appears twice in chapter 5 (V.47. V.48) and the expression "laws of supply and demand" appears 18 times dispersed in 17 paragraphs including Prefaces for the 1st and 2nd editions.
[24] Jevons cites wrong pages.

curve). Most of the time when he talks about demand changes, he explains promptly that the final degree of utility (or derivative of the total utility function) deceases according as the quantity of the commodity one possesses increases. Although there is no such expression, I have an impression that Jevons was thinking that the laws of supply and demand are equivalent to the diminishing law of marginal utility.

Jevons's account of exchange is a strange one. He emphasize that the total utility must be maximum after exchange for both two traders. He writes an equation such as

$$\varphi_1(\,a-x\,) \,/\, \psi_1(\,y\,) = y\,/\,x = \varphi_2(\,x\,) \,/\, \psi_2\,(\,b-y\,). \qquad (7\text{-}1)$$

"[W]henever two commodities are exchanged with each other, and *more or less can be given or received in infinitely small quantities*," (Jevons's emphasis) Jevons explains that the quantities exchanged *x* and *y* must satisfy these two equations. Jevons assumes that the total utility and by consequence the final utility are separable by each commodity. Functions $\varphi_1(\,s\,)$ and $\psi_1(\,t\,)$ signify trader A's final utilities when A possesses *s* units of corn and *t* units of beef. Functions $\varphi_2(\,u\,)$ and $\psi_2\,(\,v\,)$ signify trader B's final utilities when B possesses *u* units of corn and *v* units of beef. Even if the utility functions are not separable, the equations hold when we replace $\varphi_1(\,a-x\,)/\psi_1(\,y\,)$ and
$\varphi_2(\,x\,)/\psi_2\,(\,b-y\,)$ by $\partial\Phi/\partial s \,/\, \partial\Phi/\partial t$ and $\partial\Psi/\partial u \,/\, \partial\Phi\Psi/\partial v$ evaluated at ($a-x$, *y*) and at (*x*, $b-y$) respectively.

If we use Edgeworth box diagram, we can express these relations by Figure 3. The only difference between Jevons and Edgeworth lies in whether they admit the middle side y / x is equal to the left- and right-hand sides. Jevons claims that the middle side is equal to either of both-hand sides. This is a simple consequence of the law of indifference.[25] We may argue long on this law, but here is not the right place to do. Edgeworth would not admit this law. By consequence, we can say that the difference between Jevons and Edgeworth is whether they admit this law or not. Except this single point, Jevons can be said to be a forerunner of Edgeworth. We will discuss this point in the next section.

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[25] Whole of chapter 4 goes into the explanations of this law.

B's consumption of good 1

B's possession of good 2

B's consumption of good 2

A's indiffer-
ence curve

B's indiffer-
ence curve

A's consumption of good 2

y

exchange
ratio

x

A's consumption of good 1
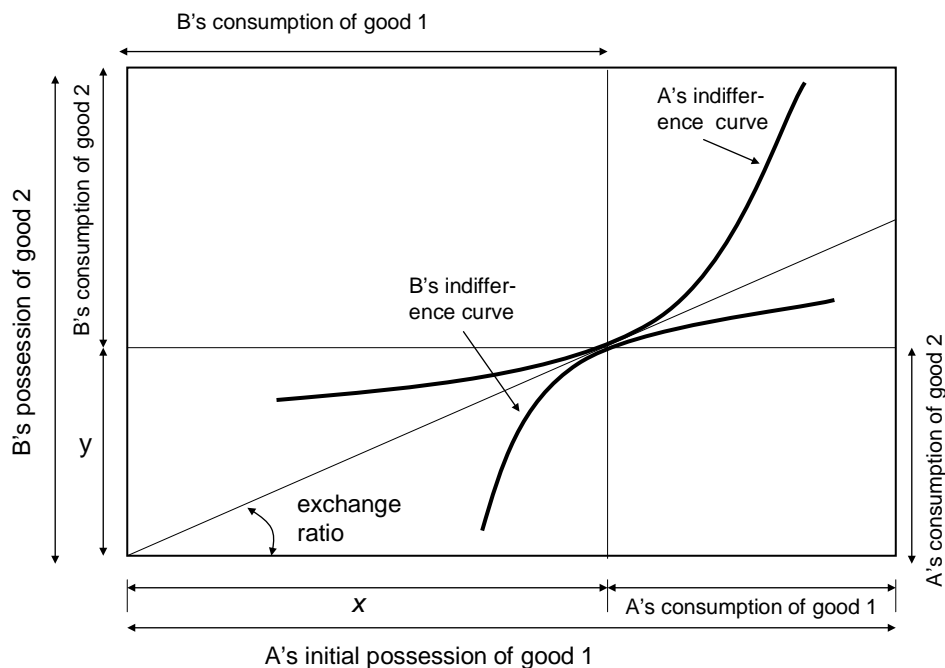
A's initial possession of good 1

Figure 3

Contrary to well established belief, laws of demand and supply do not need demand and supply functions. We may argue that this implies intermediate character of Jevons's economics. A common understanding is that Jevons failed to grasp these crucial concepts. Another strand of understanding is to situate Jevons on a way to a deeper understanding of exchange process (Fonseca and Ussher 2002; Nakano, 2009). I am rather inclined to the first understanding and I will explain the reason soon after.

If we admit the law of indifference, equation (7-1) has a good chance to have a unique solution. The cases of non-existence of solutions are not excluded. When solutions exist, they may not be unique, but in general they form separated discrete set. If we do not ask how we get one of these solutions, Jevons argument must be estimated quite satisfactory. There is no need to examine exchange process by demand and supply curves.

Fonseca and Ussher (2002) and Nakano (2009) think that there must be a route other than the scheme that comprises demand and supply functions. They are right to believe that demand and supply functions are artifacts that few empirical

supports and no deep theory basis. I do not enter into this question but those who doubt this are requested to read Erik Beinhocker (2006), Steeve Keen (2011) and my papers (Shiozawa, 1999; 2016b). However, I believe that Jevons (1871) has another aspect that Fonseca and Ussher (2002) and Nakano (2009) do not pay due attention. This is the point that Aldrich (2000) observes. Aldrich claims that Jevons's Theory of Exchange chapter is not a simple theory of (domestic) exchange, but a unified theory of both domestic and international trade.

Aldrich (2000) is right. Jevons introduces in chapter IV on Theory of Exchange a term "trading body", which stands for any trader who may be either "a single individual in one case" or "the whole inhabitants of a continent in another".(Jevons, Library IV.19) He justifies this singular terminology because "the principles of exchange are the same in nature, however wide or narrow may be the market considered." (Jevons, Library IV.20) Jevons adds that "our laws of Economics will be theoretically true in the case of "individuals, and practically true in the case of large aggregates; but the general principles will be the same, whatever the extent of the trading body considered. We shall be justified, then, in using the expression with the utmost generality." (*Thoery*, Library IV.20) Thus, Jevons's theory of exchange intends from the start to be a unified theory that is applicable both for domestic exchange and for international trade. Aldrich emphasizes its possibility in this way:

> As in the aggregative neoclassical analysis of the 1930s, international trade was conceived as exchange between countries, each with its own preferences and production possibilities. Jevons developed marginal conditions for consumption and production and extended the theory of comparative advantage in a way that did not become established in the literature until the 1930s. (Aldrich 2000 p.65)

Aldrich (2000) asks himself why Jevonian revolution did not take place in international trade theory unlike the case of domestic theory on which Jevons's agenda setting, mode of argument, and analytical tools had made a tremendous impact. In my judgment, this non-existence of a revolution is rather a natural course of the economics, because Jevons did not make any substantial development in international trade theory. Except for mathematical formulations, Jevons and Mill are treating the problem in the same framework: pure exchange economy. Moreover,

he could not treat any topics that are specific in international trade situations. For example, he could not device any account on the big disparity of real wages between developed countries and then mostly colonized underdeveloped countries. He could not explain how the international specialization takes place[26]. This was inevitable, in my opinion, because Jevons did not understand the essential difference between exchange in a country and exchange between countries and this is why he thought he could unify exchange within a country and between counties.

Aldrich (2000) posed an interesting question, but it was ill-posed, or at least it was shallow. If we go in the depth of the problem, we should ask why Jevons came to consider his pure exchange economy or an economy where we have two persons and two commodities. Let me cite Jevons's own proclamation:

> The keystone of the whole Theory of Exchange, and of the principal problems of Economics, lies in this proposition—The ratio of exchange of any two commodities will be the reciprocal of the ratio of the final degrees of utility of the quantities of commodity available for consumption after the exchange is completed. (Italic by Jevons,.Library IV.29)

This is a manifesto of his marginal utility doctrine.

> Imagine that there is one trading body possessing only corn, and another possessing only beef. It is certain that, under these circumstances, a portion of the corn may be given in exchange for a portion of the beef with a considerable increase of utility. How are we to determine at what point the exchange will cease to be beneficial? (IV.30)

This is an economy with two persons and two commodities. Where does this setting come from? When we consider exchange, we ordinarily think two-person, two-good situation. Do we do because this is the minimal? Please reflect if this kind of setting

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[26] In chapter 5 where he explains labor and production, Jevons treats international trade under the headline of Various Cases of the Theory. He argues correctly when trade ("foreign commerce" in Jevons words) is excluded. It is the case where $\omega 2/\omega 1 = \mu 2/\mu 1$ (Jevons's substitutes to Ricardo's four magic numbers) but, to study specialization, he had to analyze the case where the two ratios are not equal. (IV.42) See also the next footnote (on Turgot).

was common before Jevons. Did Adam Smith or David Ricardo start to discuss exchange from this setting? They did not. [27] Present day economists are accustomed to think with this abstract situation, but this is the very custom that the neoclassical revolution created. Smith, Ricardo and other classical economists thought of existing economy. Perhaps we may call it industrial capitalism. It is a system of division of labor where tools and machines play an important role and exchanges are made by means of monetary. An economy of two persons and two goods may take place when two persons meet by chance in the middle of a desert, one with water and the other with food. [28] This is the world that Jevons wanted to analyze. It is a possible situation but not very important for a real life. It is true that Smith talked the story of a deer and two beavers. This was but a simple illustration, not a target of analysis. What he wanted to describe and analyze was the emerging industrial capitalism. That is why he started to talk about pin making.

Jevons set an abstract economy comprised of two traders and two goods. The traders may have their own utility but no other information nor social institutions. This is a very simple situation one can imagine and is adapted to a pure analysis. How did Jevons come to make this setting? As a result of logical deduction? If we think in an abstract way, exchange must comprise at least two persons and two different goods. Then, two-person, two-good economy must be the minimal setting for analysis without any other givens except that persons in trade have their preferences and provided that they will not appeal to violence. We are accustomed to think in this way. However, Jevons himself was not thinking in this way. He wrote for example "It is also essential that the ratio of exchange between any "two persons should be known to all the others." (Jevons, Library IV.16)[29] If this is a minimal requites for a market to work, Jevons was implicitly violating his own situation setting.

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[27] Ricardo (1817) and James Mill (1922) refer to two-country, two-commodity exchange, but it was not considered as an isolated abstract economy. Reference to Robinson and Friday goes back to Bastia (1848) 6. Property and Plunder, Library 6.63. I owe this to a hint by Giulio Palermo. *Economic Sophisms* of the same author and the same year contains three sophisms on the same theme.

[28] Turgot in his unfinished paper *Value and Money* (1769) examined just this kind of situation. See Turgot (2011, pp.173-178). Murray Rothbard considers that this is the first Crusoe economics (ibid, p.xiii). It is not certain if Jevons knew Turgot's *Value and Money*. In the *Theory*, Turgot is only named as a piece of Condorcet in the list of bibliography. In Jevons (1875) Turgot appears each one time in Chapter IV and VII, but there are no mentions on the Turgot's theory of exchange.

[29] Turgot (1769) is not making such an assumption in the two person exchange case.

My hypothesis is this: John Stuart Mill's trade theory made many economists to concentrate in this abstract setting of two-country, two-commodity exchange and this triggered immergence of the theory of pure exchange economy.

This hypothesis has several circumstantial evidences:

(1) Mill's *Principle* (1847) was the most influential economics textbook at the 3rd quarter of the 19th century in the United Kingdom.

(2) Mill advocated a reversion from Ricardo's cost-of-production theory of value to more fundamental law of demand and supply.

(3) The structure of Mill's economics is composed of two theories of values, one domestic and other international and despite of Mill's claim[30] theory of value was not sufficiently unified.

(3) It was normal for ambitious economists in 1860's to want to build a unified theory of economics (or value theory) and Mill's example provided almost unique important case to start their trials.[31]

(4) International trade was a paramount situation which was too important to be treated as exceptions and required a new value theory.

(5) Despite of his general hostile, contending attitude with respect to John S. Mill, Jevons is particularly conciliatory to Mill's trade theory.[32]

What is important here is not how Jevons himself thought consciously. Jevons did a great leap forward in economics. He changed the economics from Plutology

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[30] Mill (1848) claimed that "there is nothing in the laws of value which remains ... to clear up" (Library III.1.2). It is a mystery why Mill put this self-betraying proclamation.
[31] Mill (1848) cited the "joint cost of production" case as a reason to revert from cost of production theory to more fundamental law of demand and supply (Mill, 1848, Library III.16), but this is not comparable to the international trade case by their importance.
[32] Jevons praised Mill's theory of international trade as "ingenious" and "nearly always true." (Library IV.100). He also mentioned that he thoroughly concurs with his citation from Mill (1848): "Almost every speculation respecting the economical interests of a society thus constituted, implies some theory of Value" (Library IV.2)

(economics of production) to Catallactics (economics of exchange). His book *Theory of Political Economy* proves it. Characteristically, production comes at chapter 5 after chapter 4 which deals with theory of exchange. Chapter 5 is titled Theory of Labour and there is no chapter with production in the title. In the Preface to the first edition, he spoke of exchange but no word "production". Jevons made his revolution unconsciously. He did not make his catallactic revolution not on the basis of deep study of economics of production. Even when he talks about production, it is a production which does not suggests industrial capitalism. Workers he refers are not employed workers but better interpreted as self-employed ones.

On the conscious level, Jevons was decisively influenced by two ideas and he knew it. One was the idea to apply mathematics to society. Another was the idea of utility. The two were combined to coin a new idea, "final utility" or marginal utility in a more standard terminology. Some economists praise that this was really revolutionary. I do not think so. If the idea of "utility function" might be new, the conception that utility determines the prices of goods was as old as economic thinking. Classical economics immerged from the *rejection* of this "common sense." Jevons did not understand this crucial point. [33] Except for his adoption of mathematical formulations, as Marshall (1872) pointed, Jevons's *Theory of Political Economy* contains no substantially new propositions.

Jevons made a revolution, or more exactly, a counter-revolution, but he was not aware of this fact. Then, from where did this change come? If my conjecture is correct, he came to know Mill's international values chapter and he got an idea to construct whole economics on the basis of international trade situation. If this is the case, it is normal that Jevons emphasize that there is no difference of logic between domestic and international economics, because in his understanding his new theory was a generalization of international exchange.

Aldrich (2000) had a good insight, but his understanding was totally wrong. Jevons

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[33] If Jevons claimed that the final utilities of two newly purchased goods are the same, he was right, but this does not mean that final utility determines the values of goods. If goods are produced as much as they are demanded at the price set by producers, it is this price which determines the actual price. Final utility selects those who estimate it higher than the final utility of the payable money in exchange of the product. This does not determine the price but who buys the product at that price.

did not succeed in constructing a theory of international values. He claimed so, but that does not mean that he really did. He may have built a theory of pure exchange. It may be applicable to the long distance trade between different communities but it is not a theory applicable to an industrial economy. The lack of notions such as specialization and comparative costs is the evidence of his failure. In opposition to Aldrich's expectation, it was inevitable that neoclassical revolution in the theory of international trade did not take place long after Jevons. Aldrich (2000) assumes that this delayed revolution arrived in 1930's, but it is no place here to trace theoretical development of the "modern theory" of international trade (Chipman, 1969).[34] We are more concerned with the neoclassical revolution and for this purpose we cannot dispense with two more grand names. One is Edgeworth and the other is Marshall. In the next section we discuss Edgeworth and in section 10 we discuss Marshall. Both of them were heavily influenced by Mill's international value chapter but in a very different way.

§9. Francis Ysidro Edgeworth

Although the Edgeworth was born and dead about 2 years later than Marshal and the former expresses intellectual obligation to the latter, I put Edgeworth before Marshall, because (1) Edgeworth and Marshall stand on opposite extreme and (2) Edgeworth has more common points with Jevons than Marshall.

In the previous section, I produced Figure 3 as an illustration of Jevons's theory of exchange. The situations they set are quite similar. Namely, both are two-person, two-commodity exchange case. Similarity of Figure 3 and Edgeworth's idea are apparent if once we use later named Edgeworth diagram.[35] Of course, there are some differences. Most important point is that Edgeworth did not admit that y/x (the middle side of (7-1)) is equal to $\varphi_1(a-x) / \psi_1(y) = \varphi_2(x) / \psi_2 (b-y)$. In the diagram, this difference is reflected whether the common tangent line passes through the origin. Jevons thought that it does and Edgeworth thought not necessarily it does.

This difference reflected two economists' view on exchange process. Jevons's exchange is *mediated* by prices (but not by money). In other words, when exchange

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[34] As for internationa trade theory Tabuchi (2017, in this volume) gives an interesting account.

[35] Creedy (1992) emphasizes the same point.

takes place, Jevons assumed that two persons change their possession in such a way that the value one receives is equal to the value one renounces by mutually agreed prices. Edgeworth is much more radical thinker of exchange. He wants to analyze a pure exchange situation before the notion of price is established between people. Edgeworth assumes an extremely pure exchange economy. Notions such as prices are something invented by somebody in very old days and institutionalized after they were repeated experience of the people. By rejecting these preconceptions, Edgeworth wanted to put his theory of exchange on a firm basis.

I am not sure if he has succeeded in this project. *Mathematical Psychics* is one of Edgeworth's earliest and decidedly his major works. In this book, Edgeworth defines field of competition as an exchange economy among indefinite number of individuals. A field of competition is defined to be perfect when 4 conditions of free contract are satisfied. In this field of competition, a *settlement* is "a contract which cannot be varied with[out?: Shiozawa] the consent of all the parties to it," a *final settlement* is "a settlement which cannot be varied by recontract within the field of competition," and finally "[c]ontract is indeterminate when there are an indefinite number of final settlements." (Edgeworth 1881, p.19). The intent of introducing these concepts, after him, is to investigate "How far contract is indeterminate." (ibid. p.20)

To begin with, Edgeworth starts from examining Jevons's two-trader, two-commodity case. As he does not admit Jevons's law of indifference, the common tangent curve expressed by $\varphi 1(a - x) / \psi 1(y) = \varphi 2(x) / \psi 2 (b - y)$ is not necessarily equal to y / x.
Lacking the middle side of (7-1), Edgeworth's condition does not determine an exchange rate, whereas Jevons's conditions normally determine a solution. In the later analysis, Edgeworth goes on to examine the cases of "several persons and several variables"(p.26). Jevons could not treat these cases, but the extension was not difficult for Edgeworth, because he knew how to use Lagrange multipliers. Edgeworth even introduces the concept of Pareto efficiency by the term "relative maximum." (p.23).

Mathematic Psychics is full of sinuous arguments and difficult to grasp his real contention. It seems, however, that Edeworth was unsatisfied by the treatment of exchange process by Jevons, Marshall and Walras. I cite only those names that

Edgeworth named explicitly. These authors assume a price prevailing in a market and appeal to a concept of aggregate quantity demand. For Edgeworth, such a treatment is not at least general in view of haggling deployed in the market. He explains the superiority of his method citing that his theory is applicable to the cases of imperfect competition.

Edgeworth's idea was buried soon after him and "remained dormant until it was ressureccted by Martin Shubik (1952) as the theory of 'core'." (Fonceca, XXX, Edgeworth's "Indeterminacy of contract", 1. Introduction). The literature exploded in 1960's and Edgeworth's three conjectures are proved (Debreu and Scarf 1963). Edgewoth's conjectures are now theorems. Citing this fact, many historians of economic analysis now remark on Edgeorth' s method. Some scholars like Fonseca and Ussher (2002), Donzelli (2007; 2009) and Nakano (2009) claims that there were another route of development other than that of Marshal and Walras, or economics based on the concept of demand and supply functions.

I heartily admit this possibility, but I doubt if it can be a realistic economics. The concept of core assumes that a solution is not blocked by any coalition of the set of traders in the market. Let it be a set of N traders. Then the number of possible coalition explodes by the order of 2^N. This means an explosion of information and communication. Edgeworth's set of final settlements may shrink to Walrasian equilibrium without checking all coalitions. Suppose that the interventions of a third party are sufficient for the shrinking. Even in such a case, the information that should be communicated between traders is exorbitant. It would not provide an exchange system that permits to run an economy as big as a small nation. It would be better to interpret Edgeworth's idea as an illustration of multi-partite bargaining process. In this regard, I believe that Marshall's treatment is much more effective and realistic, although I do not adhere to any demand and supply cross-point parable. I will pick up the Marshall case in the next section and discuss problems of his formula. However, our main concern is not the possible future of the theory. Our main object is to know where Edgeworth's idea came from.

In *Mathematic Psyhics* we find few mentions on Mill (two paragraphs). In a later long article in *Economic Journal*, of which Edgeworth was the editor, Edgeworth (1894-85) repeatedly mentions John Stuart Mill and his International Value chapter, calling it "great chapter" (3 times), "stupendous chapter" (2 times), and "classical

chapter" (1 time). The reason why Edgeworth did not argue Mill's international value chapter is unknown. He may have got his idea from Jevons (1871) or Marshall (1879). Although his main example was two-trader, two-good case (he repeatedly cited Robinson and Friday as a case of isolated contract), he knew how to deal with many-trader, many- good case in an appropriate way. Despite of all these facts, I think I can claim that Edgeworth was also influenced by Mill's international value chapter in a deep level.

First observation is that he thought a pure exchange economy without any production. Such a situation only exists in the international value chapter in Mill (1848). Second, when he reflects on The Theory of International Values, Edgewroth (1884-85) considers two-country, three-commodity case. In such a case, he had two options: (1) complete specialization case (a country produces two commodities and another country produces the other third commodity, and (2) partially specialized case where two country have one commonly produced. In the second case, as Edgeworth discusses himself in Part III, the relative prices of all commodities are determined through this common good. Why did he give priority to the first option?. Isn't this a symptom? Third, Edgeworth's emphasis on indeterminacy is very unique among economists. If we search such a case in the texts before Edgeworth(1881), Mill (1844 First question) and Mill (1848, III.18) immerges as probable source of inspiration.

As I have written above, Edgeworth is doubly influenced by Jevons and Marshall, it is difficult to detect a direct evidence, but we can at least say that he was not out of John Mill's field of problem setting. Standing on Mill's problem setting, Edgeworth developed a theory of exchange toward an extreme opposite. If we combine this fact with Marshall's direct influence from Mill, we may claim that Mill's proposal to revert to more fundamental and ancillary law compelled the next generation after him to an economics of a wide variety but within a definite range of directions.

§10. Alfred Marshall

The case of Marshall is much easier than Jevons and Edgeworth, because we have clear evidence. In fact, he wrote Marshall (1879) which was never officially published in his time but distributed privately by the hand of Sidgwick. The date 1879 stands for the year of this private publishing. This was a part of a two-volume

book that was originally planned to be *Outline of Political Economy* which started to be composed between 1872 and 1874 (Whitaker 1975, I. 260). Jevons (1871) was published before this, but if Marshall's recollection is right, he was not much impressed by Jevons' book (Marshall 1925, appendix to review of Jevons (1871)). It will require a scrupulous study of Marshall's theoretical development to discern influences of John Stuart Mill on different themes, but in this paper I concentrate to the examination of two *Pure Theories* which were intended to compose a unified book.

Although he has made some contrivances, Marshall's *Pure Theory of Foreign Trade* is a kind of mathematical annotation of Mill's international value chapter. Of course, he is much clear than Mill and successful in providing a proof of uniqueness (Proposition VIII). From a modern mathematical view his explanation does not pay attention to existence problem, even though a sufficient condition is already given by John Stuart Mill. As many economists mention it, Marshall started to consider stability and instability of equilibrium. It can be connected at least to cob-web theory of the 20th century. It is still questionable if his treatment of stability really dynamic or not, but our main point of investigation is not here.

Our main interest is how Marshall was influenced by Mill's treatment and argument of international trade. The direct influence of Mill' international value chapter on Marshall's *Pure Theory* is so evident that we have no necessity to prove it. The question is how Marshall reformulated Mill's problem. A conspicuous fact is that Marshall has invented the concept of demand function. Simply speaking, Jevons did not have a concept of demand function (See section 8 of this chapter). Edgeworth, having read Marshsll's *Pure Theory*, criticized that concept. Consequently, Marshall was the unique person among British founders of neoclassical school, who put the demand function at the core of economic analysis.[36] More subtle question is why he did not use the expression "supply curve." In the case of foreign trade case (in Mill's setting), there is no difference between demand and supply curve. As they came to be called "offer curve" by later authors, the same relation is called as demand or supply by the difference of point-of-view. In the case of *The Pure Theory*

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[36] Marshall (1879) contains the term "demand curve" but no "demand function." But we assume that "demand curve" is employed to express demand function in the case of two goods case. As he has adopted an exchange between a good and money, any functions can be called simply "curve". Yoshii (2017 in this volume) examines Jenkins' contribution to the formation of demand function concept.

Of (*Domestic*) *Values*, the companion volume of the *Outline*, two expressions demand curve and supply curve appear almost as frequent as the other. The natural question is, then, this: did Marshall have a chance to invent demand and supply curves in other circumstances than he examined Mill's international value chapter?

We have no direct evidence that prove this. Instead, we have some indirect evidence that Marshall got the idea of demand and supply function in the course of studying theory of international values. There are three circumstances for this thesis, although each of them is closely connected with each other.

The first circumstance is the special composition of the *Outline of Political Economy*. As indicated above, this unfinished book contains two major parts: The Pure theory of Foreign Trade and The Pure Theory of Domestic Values. Contrast between domestic and international still continues in our days. The particularity of Outline is that the volume on foreign trade precedes that of domestic values. This is quite strange structure in our days. Normally we first study theory of domestic economy and then proceeds to foreign economy. Marshall has taken an opposite composition and we should ask why.

This strange composition seems to reflect Marshall's process of theory construction. At the beginning of the *Domestic Values*, Marshall criticizes Mill's usage of the term "theory of value". In Marshall's idea, "theory of value" must be a generic term which should include both of domestic and international values. He is right but why did he put theory of international values before the theory of domestic values? Marshal justifies this by saying that "[t]he apparatus of diagrams which was best adapted for the investigation of the latter will not be of service here [in theory of domestic values]".(p.1) The "apparatus of diagrams" is apparently the diagram in which two demand curves (or in the more modern terms two offer curves) cross. This hints that so-called Marshallian cross came first in his mind when he was considering international values and the domestic version was completed after that.[37]

The second circumstance is the argument and reason that Marshall gives why the theory of domestic values is more difficult than that of international values. Most of

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[37] Sraffa (1926)'s criticism on Marhall's price theory stats form pointing its fundamental symmetry. He remarked that this is not an old tradition.

present-day economists must think in an opposite way, but Marshall thinks differently. In the theory of international values, the demand curves of one country is the supply curve for the other country. In the theory of domestic values, Mashall warns that there is no such symmetry between demand and supply curve. It is Marshall who has established the logical symmetry of demand and supply curves. In this particular point, he is right. Marshall's situation setting in the *Pure Theory of Foreign Trade* is quite ambiguous, because he contends that he is thinking a case of many commodities. But, it is clear that he is thinking a situation where a pattern of specialization is already determined. If the bundle of commodities that is produced in a nation has predetermined proportions, the amount of production of these bundles is uniquely determined. Then, he is in fact considering a pure exchange economy. In the theory of domestic values, he adopts money as the commodity to be exchanged against a commodity to be considered. This presupposes an economy that Edgeworth would prefer to avoid. Even if Marshall assumes commodity money, the definition of supply curve must have been no easy attempt.

Perhaps we may conjecture that this introduction of supply curve is Marshall's major contribution and the second crucial bifurcation point between classical and neoclassical economics.[38] Marshall may have assumed that he had defined supply curve in a satisfactory way. He may have been dissatisfied by his clumsy and in fact confused definition of supply curve. We cannot tell firmly which the case was. If it is the second case, this may have been the reason why he could not complete the *Outline*. If it is the first case, he was in fact trapped in a logical error. It is also possible that he wondered between the first and second state of mind.

Marshall's definition of supply curve is based on what he calls "the great central law of economic science." (*Domestic Values*, p.3, Whitacker p.188) Citing long a part of an article[39] he has published in 1867, he explains as follows:

> This law is [1] that "producers, each governed under the sway of free competition by calculations of his own interest, will endeavour so to regulate the amount of any commodity which is produced for a given market during a

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[38] The first bifurcation point was, as I have explained above, the Mill's choice as standard situation of international trade situation the Mill-Jones point of two-country, two-commodity economy.

[39] "Mill's Theory of Value" that appeared in *Fortnightly Review*, April 1876. Reprinted in Pigou (1925), Part II, Chapter 3, pp.119-133.

given period, [2] that this amount shall be just capable on the average of finding purchasers during this period at a remunerative price. [3] A remunerative price is to be interpreted to be a price which shall be just equal to the sum of the exchange or economic measures of those efforts and sacrifices which are required for the production of the commodity when the amount in question is produced. [4] These economic measures are the expenses which must be incurred by a person who would purchase the performance of these efforts and sacrifices." (Marshall 1925, pp.126-127; Whitacker 2 p.188. Square brackets are inserted by Shiozawa.)

Recall that the cited part is the Marshall's summery of what he thinks as the central truth of the Political Economy. When we put this citation in the original text, we see that "the central law of economic science" expresses an essence of Mill's *Principles of Political Economy*. Of course, Marshall keeps some reserve such as a proposal to replace "cost of production" by an expression like "expenses of production," which is he believes more protected to misunderstanding.

The question we have to examine here is the fact that Marshall uses the term "law" in singular form. In the citation, many different propositions are in fact told. Is this a single law? Or, is it a composition of several different laws? Part [3] explains the meaning of the term "remunerable prices". Part [4] is related to correction or more exact interpretation of the cost of production. For a long time, cost of production was commonly interpreted in real terms as "those efforts and sacrifices which are required for the production of the commodity when the amount in question is produced." Marshall corrects this common interpretation. It should be understood as the sum of expenses required for the production.[40] However, this central law comprises at least two different ideas. Part [1] states that producers regulate the amount of their production for a given market during a given period. Part [2] states that the produced goods are sold at a remunerable price.

If we understand this law as a law of classical economics, part [2] means that commodities are sold at a remunerable price in average. The classical theory of value assumes that the value is determined by the cost of production (full cost in

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[40] In spite of this remark, the real cost interpretation continues until 1930's at the time of Viner. In later part of the 1876 article, Marshall himself was contradicting himself. At least, he admitted that he had not succeeded in unifying two theories of value, domestic and international.

the modern terms). It may fluctuate by the discrepancies of demand and supply, but comes back to the natural price or value soon or later. Producers assume the quantity of demand that will be sold at this natural price and regulate the quantity of production. The adjustment process may be complicated but in average the price of a product moves around the natural price and the production is regulated so as not to increase the discrepancy between demand and supply too big. This is the central truth of the political economy that Marshall interpreted. But, is this the same law that Marshall formulated by his demand and supply curves?

John Stuart Mill characterized as "a law of value, anterior to the cost of production, and more fundamental"? (Mill, Library III.16.5)

By the law of demand and supply, many economists will now imagine that a producer chooses an amount of his/her production for any price that is given in the market. The above central truth tells no such things. Fundamental assumption is that prices fluctuate around the natural price. If the market price leaves far from the natural price, the producers would not know and cannot calculate the optimal quantity at that market price. They only increase their production if the market price is higher than the natural price and decrease in the opposite case. This is a similar process that Leijonhufvud (1968) called *cybernetic approach* to macroeconomics. The producer has no definite quantity to produce that is best for him or her. I will argue this point soon after.

A misinterpretation must have occurred in Marshall, because he induces the existence of supply curve form this central law[41]. The formal definition of supply curve is given as follows:

> The Supply curve *SS* for a commodity in a market is such that if any point P_2 be taken on it, and P_2M_2 drawn perpendicular to *Ox*, P_2M_2 represents the price per unit at which a supply of the commodity of which the amount is represented by *OM₂* can be remuneratively produced and brought into the market in each year (or other given period). (Sidgwick, 1879, p. 5 ; Whitaker 1975, p.192)

In order that this definition is valid, as I will show it soon later, we need an important

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[41] Yoshii (2017) examines the same point from a different angle.

condition: the law of decreasing returns. He may know this fact. Just after giving the formal definition, Marshall states that "It may be that every increase in the amount supplied involves a more than proportional increase in the expense of producing it." However, he also talks about increasing returns to scale case in the same section. In contrast to a "raw commodity" which we can assume to obey the law of decreasing returns, Marshall explains that, in cases of most manufactured commodities, the law of increasing returns holds. (Sidgwick, 1879, p.6; Whitaker 1975, p.192) How can we define the supply curve in these cases? We cannot.

Let me explain it in more modern terms. It is supposed that producers were to regulate the amount of their product for a given period of time and at a given price system. This means that they prefer that amount of production in comparison to all other amounts. How can this happen? The only possible situation is that they are maximizing their profit at an amount of production. The concept of supply curve implies this. Then, for the profit maximization to hold, the production must be at a decreasing returns to scale point. In fact, let $f(x)$ be the total cost function when an amount x is produced. The profit is given by

$$p x - f(x) \qquad\qquad (9\text{-}1)$$

under the assumption that all products are sold at price p. If the profit attains maximum at point x, expression (9-1) must have the derivative 0. In other words,

$$p - f'(x) = 0. \qquad\qquad (9\text{-}2)$$

If the product has a positive price p, the marginal cost $m(x) = f'(x)$ must be positive. If the firm is operating with profit, this also means that the return is decreasing to scale. Bt definition decreasing returns to scale means that the average cost is increasing. This is easy to confirm. In fact, let $a(x)$ be the average cost, that is the total expenses divided by the amount of the product and take a derivative of function $a(x)$, then

$$a'(x) = \{f(x)/x\}' = \{f'(x)x - f(x)\} / x^2 = \{p - a(x)\} / x > 0 ,$$

when the profit is positive and x satisfies (9-2).

The contrapositive of the above observation is the following: when the returns to scale is increasing or constant, supply curve as a function of the product price has no proper meaning. It is not clear if Marshall was well aware of this fact. In the *Pure Theory of Domestic Values* no explicit mention is found. Marshall cites cases of increasing returns but makes no warnings.

Another symptom that Marshall is confusing is that he often indentifies the supply curve and the expense curve (or the total cost function). We can define expense function without any condition, but the expense function and the (inverse function of the) supply function do not coincide unless the return to scale is decreasing.[42] If the average cost function or is constant or decreasing, we cannot define the desired supply level of productions.[43]

Marshall passed over the difficulty of defining supply curve. This point remained the source of trouble through out the Marshall's life. In the *Principles of Economics*, he evaded the question by introducing the distinction of internal and external economy. It was still at the early stage of giant firms. However, with the lapse of time, it became clear that majority of firms face increasing returns to scale. It was just after Marshall died that Piero Sraffa (1926) wrote a famous paper On Laws of Returns and pointed that the production was not limited by the increase of costs but by the limit of sales. This is simple denial not only of Marshallian cross framework but also of the general equilibrium framework. However, at this time, neoclassical economics was too firmly established. No reformulation was made within the neoclassical economics and it now faces the same difficulty when Sraffa argued law of returns.[44]

We can now understand the real nature of the problem. Marshall wanted to build a theory of domestic values on the same principles as the theory of international values. In the *Pure Theory of Foreign Trade*, Marshall successfully reformulated John Mill's trade theory to a more formal symmetric theory of reciprocal demand curves. He knew there was no similar symmetry in the domestic economy and he introduced the concept of supply curve. It was a natural reaction to the theoretical situation that Marshall faced. He did not know the real trouble that the notion of a supply curve implies. He must have been driven by the charm of re-establishing the symmetry of supply and demand. When we view from our common knowledge of the present, Marshall's choice may not seem a special decision. If we know, however, the real trouble of the concept of supply curve, we see how deeply

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[42] This misidentification partly explains why Marshall employed the quantity (instead of price) as horizontal axe whereas he supposes that prices are given. Expense curve can be defined for any production quantity, if that production is possible.

[43] This is the major reason why neoclassical economics prefer to assume decreasing returns to scale albeit it is rare to find such an industry or firms. See Shiozawa(1999; 2004; 2016b).

[44] For more details, see Shiozawa (2016b), sections 2 and 3.

Marshall was oriented by the Mill's thesis on the necessity to revert to the law of supply and demand.

The third circumstance is the introduction of the concept of "consumers' rent". In *Pure Theory of Domestic Values*, the term "consumers' surplus" is not yet used. If we read the definition of the term "consumers' rent" or the mathematical footnote at the end of section 3 of chapter 2, we can easily know that this is an antecedent of the later "consumers' surplus." Why did Marshall feel it necessary to introduce this concept? A simple answer is: to make in domestic economy a theory that is comparable to that of the gains from trade. I have no space to argue this point in detail. This must be one of three circumstantial evidences that Marshall was deeply influenced when he started to build his new theory of economics.

At the end of discussion on Marshall, let us make a brief comparison between Edgeworth and Marshall. In a word, Edgeworth was more inclined to pure logic of exchange and Marshall was more realistic. Although Marshall's concept of supply curve has a serious defect, his idealization was based on monetary economy. Edgeworth wanted to exclude it. He was more loyal to the idea of catallactics. However, he could not really think how the large network of exchanges can actually be organized. If we think of this point, money and price system (including price-mediated exchange) are indispensable institution that makes modern large-scale economy work. As the trouble with neoclassical economics is deepening, there are some reflective movements in search of alternatives. It is true that Edgeworth show an extreme opposite to Marshallian paradigm. But the search for a pure theory of exchange is itself a product of Mill's misguiding problem setting. It is necessary to return before the Mill's solution and his reversion.

§11. Alternative way outs? / Senior, Mangold and Sidgwick

We have traced the history how Mill's "solution" paved the way to the neoclassical revolution. It may seem that the force that led this history was overwhelming and there were no alternatives to this trend. But we may ask if there was no way out. Were there no trials in the direction of a new theory? Yes, of course, there were. McKenzie's Princeton teacher Frank D. Graham was the most famous dissident to the mainstream tradition. Even before Marshall and Edgeworth, there were three notable exceptions: Senior (1830), Mangoldt (1863) and Sidgwick ([1883] 1901).

Latter two cases are reported in Edgeworth's survey (189-95, III).

I do not explain in detail. Edgeworth (1895) has given a good concise account of two contenders (A.4; B.3). Magoldt (1863) has written in German and his influence to the English world was limited. He was somewhat forgotten economist even in German world (Schneider, 1960). Jevons (1871)'s second edition includes the book in the list in appendix but no explicit mention on him in the Preface to the Second edition, where Jevons tried to explore all important precursors in mathematical economics. Marshall once cited Mangoldt's name in his *Principles*, but no mention appears in his correspondences (Schneider, 1960). Without Edgeworth's subsection on him, he may have been forgotten almost eternally. Although we have now English translation of Appendix II (Chipman , 1975), let me pass through him and introduce Sidgwick's argument:

> [L]et us suppose that there is at least one other commodity ---say corn --- which is produced both of value, discussed in the preceding chapter, the relative values of cloth and corn in England must be determined by their comparative costs of production; and, again, the relative values of wine and corn in Spain must be determined in the same way.   (Sidgwick, 3rd ed[1891], p.213, Book II. Chapt III., section 2.)

As he suppose that England exports cloth to Spain and Spain exports wine to England, this is two-country, many-commodity economy.[45] If there exists a third commodity that is competitively produced in both countries, the (relative) wages of two countries are determined by the corn producing industry. Then, through this link commodity (Graham's term), the prices of all commodities are determined.

Sidgwick's contention can be paraphrased as follows. In a two-country, many-commodity economy, suppose there is a commodity that is produced in two countries E and S. Let w_E and w_S be the wage rates of countries E and S. If the transportation cost is negligible, we can write a price equality condition for the common product:

$$(1+\pi_E) \, w_E / c_E = (1+\pi_S) \, w_S / c_S , \qquad\qquad (10\text{-}1)$$

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[45] In the Sidgwick's expression, many commodities other than corn may exist. In this sense, he is thinking two-country, many-commodity economy, but we may represent it by two-country, three-commodity economy. See Figure 2.

Here $\pi E, \pi S$ and cE, cS are respectively the standard profit rate and labor productivity for each country. If we suppose that the profit rates of the two countries do not much differ, we have approximately

$$wE \, / \, wS \;\; = cE \, / \, cS. \qquad\qquad (10\text{-}2)$$

In other words, wage rates ratio is proportional to the labor productivity ratio. If the wage rate of a country is determined, the prices of all other commodities that are produced in the country are also determined. As the wage rate ratio is determined by equation (10-2), the prices of all commodities are determined. The cost of production theory of value holds.

The difference between John S. Mill and Sidgwick is whether they suppose commonly produced good. Graham called this common product link commodity. A simple existence of link commodities thus resurrects the cost of production theory of value à la Ricardo.

Sidgwick first published his *Principles* in 1883, a few years after he published Marshall's *Pure Theory of Foreign Trade* privately. A curious question arises. Did Sidgwick have this idea when he published Marshall's *Pure Theory*? Sidgwick used the book as text for his lecture in Cambridge. Is it in the course of his teaching *Pure Theory* that he got the idea that he exposes in his *Principles*? A second interesting question is how Marshall reacted to Sidgwick's opinion. We know that he did not complete his *Outline* which comprises two *Pure Theory* volumes. Is this the reason that he could not complete it? If so, then, why did he continue to employ the idea of demand and supply curves at the core center of his theory?

Naturally, the debate that followed Sidgwick (1983) was concerned with the existence of link commodity. In the third edition (1891, p.213, footnote 1) Sidgwick cites Bastable who has criticized this point. Bastable's critique is more apologetic than analytic, because he appeals to the authority of John S. Mill. Citing Bastable, Edgeworth (1894-59, III) argues also against Sidgwick but I do not think it is persuasive enough. In the subsection presenting Mangoldt, Edgeworth questions the possibility of coincidence that two countries have the same cost of production for a product. However, he is forgetting the possibility of a mechanism that brings them coincide, like present day fluctuating exchange rate regime. Of course, all of them, including Mill, Mangoldt, Bastable, and Edgeworth, concentrate on terms of trade and do not analyze how the demand and production may come to equalize.

For this analysis, we should know the world production possibility set. If Sidgwick and Edgeworth knew the shape of the production possibility set, for example if they had a rough idea of Figure 2, the course of argument may have been very different. If they knew Figure 2 and the reason why Mill-Jones point does not appear, they may have trekked a different historical path.

After all, Sidgwick's contention was a complete refutation of Mill's "solution" and all other followers. This is in total contradiction with what Edgeworth think the fundamental principle of international trade. This principle, according to him, implies in its negative form, that the value of articles in international market is not proportioned to the cost.(Edgweorth 1894 I, p.36) and this was the starting point of all economics of exchange programs. Edgeworth and Marshall were divided in their orientations but they could not accept Sidgwick, because it implies the fall down of all of their contributions to economics. The only defense possible for Edgeworth was to point the implausibility of the existence of such commonly produced commodity. In face of Sidgwick and Mangoldt, Edgeworth tried obstinately to defend the framework of exchange economy. This was a natural result of a revolution of economics and Edgeworth was too deeply absorbed in this revolution to convert to a new (or more classical) interpretation.

An interesting question is how Mangoldt and Sidgwick reacted to their own discoveries. If they think the true significance of their economy, they might have a chance to open a path to a totally different economics than that based of demand and supply function.

The case of Nassau W. Senior (1830) is more difficult to interpret, but maybe more interesting as an alternative to Mill's "solution". He left a series of lectures. They contain an interesting style of argument. He was more interested in the particular question such as how the value of money is determined. Consequently, he did not argue explicitly the theory of international values as it is commonly understood. Nor he did not write a textbook that explains his system of economics.[46] We have to guess through his explanations on other topics the theory that he might have possessed in his brain.

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[46] Although he talks much about wages and profits, Senior's *Political Economy* (1850) contains few discussions on how the prices are determined.

His lecture on the value of money is commonly understood as cost-of-production theory. I cannot tell if it is essentially different from Hume and others' species-flow mechanisms. Only thing I can say here is that Senior had an image that the same commodities are produced and exported and these common commodities contribute to determine the wage rate disparity between countries. Let me cite a short paragraph:

> Or to use a still more concise expression, that labour in England is eight times as productive of exportable commodities as in Hindostan, and labour in North America is one-forth more productive of exportable commodities than in England. (Senior, 1830, On Cost of Obtaining Money, p.12)

Senior points that the price of exported commodities depends on "the amount of the wages which has been paid, and the time for which they have been advanced" (ibid., p.14). If we combine these propositions, we can make a similar equation like (10-1) and the wage rate disparity follows from (10-2). If the relative wage rates are known, it is possible to tell the prices of all other commodities, because they pay the same wage rate in the same country.

By his explicit reference to labor productivity and wage rate and his firm framework of const-of-production theory of value, Senior had a chance to develop totally different theory of international values.[47] In fact, in view of the new theory of international values presented in chapter XX, almost of all Senior's arguments can be translated into the new theory.

Senior was 18 yeas junior to Ricardo and 15 years senior to John Stuart Mill. Senior's lectures in Oxford were delivered in 1820's and some of them was published between 1828 and 1830. John Mill had a chance know the Senior's cost-of-production theory of international values before Mill published Mill (1844) and Mill (1848). Indeed, as Bowley (1937) put it, Senior, Torrens and J.S. Mill

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[47] Bowley (1937, chapter 6, p.201) points the possibility of two different ways of treating international trade: one that investigates comparative physical costs and the other that analyzes in money terms. Bowley places Ricardo, J.S. Mill, Taussig, Marshall and Haberler in the first group and Senior and Ohlin in the second group. My understanding is 90 degree different from her classification. By their theories of value, Senior, Mangoldt and Sidgwick are much closer to Ricardo, because they all thought that cost of production theory of value (or classical theory of value) is valid in international trade as well as in domestic exchange.

exchanged their opinions on the terms of trade:

> The famous controversy between Senior and Torrens on the terms of trade, which was take up again by J.S. Mill, turned exactly on this question of the relevance of an analysis confined to two commodities and two countries. (Bowley, 1937, p.225)

If John Stuart Mill reflected more closely on his "solution" and study Senior, he may not have advanced that famous thesis that appealed the reversion to the more fundamental law of demand and supply.

§12. What was the neoclassical revolution? Implications for Future Research

This paper has shown an origin of the neoclassical revolution. It goes back to young John Stuart Mill, when he tried to solve an unsettled problem. It was a question left by Ricardo in the filed of international trade.

What Mill posed to himself was how the advantage of trade is divided among trading nations. He intended to solve this problem by providing a theory of international values. The simplest situation setting was two-country, two-commodity case. Nobody doubted there was a deep trap in this situation setting. Mill simply excluded the situations where one country cannot enjoy gains from trade. By this pure inference, he was lead to examine a pure exchange economy. As a result of his examination, a conclusion came to his mind. It was his famous thesis: "we must revert to a principle anterior to that of cost of production, and from which this last flows as a consequence,—namely, the principle of demand and supply." (Mill, 1844, I.19)

This thesis made a long strong influence on the research programs of economics. In the UK, three founding fathers were deeply influenced by the thesis and by the setting. They worked different ways and constructed their own economics. Among the three, it was Alfred Marshall with his demand and supply functions that paved the way to today's mainstream economics.

The neoclassical revolution in the UK was a turn from economics of production to that of exchange. Nearly 150 years after, we have trouble with economics. Many economists and non-economists now recognize that economics needs a change. What will be the remedy for that? A series of modifications will not be helpful. A fundamental re-building is required. History of the neoclassical revolution is illustrative. Neoclassical economics is a kind of Ptolemaic geocentric system. It had developed much and explains many, but it is basically wrong. It needs a Copernican revolution.

This chapter described a story. A question which was not hitherto referred to is this: why wasn't the story conceived in the past? One of possible answers seems to lie in a Takashi Negishi's postcard to the author[48] It reads:

> In our country international economists are not interested in the history of economic doctrines and historians of economic thought know little about the international economics. (Postcard dated July 8, 2011. Translated from Japanese by the author)

Professor Negishi talks about intellectual situation in Japan, but similar situation is observed in many other countries. If a tipping point of economics was in the international trade theory, a coincidence of history and theory was necessary. I am rather a theoretician and not a historian. It was Yasuaki Tsukamoto who taught me that history is a strong tool to persuade people to a new theory. He made me consider the meaning of the John Stuart Mill's "solution" in a historical perspective. He thus helped me to spot the point where the classical economics turned to neoclassical economics.

Acknowledgment

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[48] I reproduce this sentence on Negishi's permission (post card dated at December 19, 2014).

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